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Economics

A Piecewise Linear Model of Credit Traps and Credit Cycles: A Complete Characterization

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Introduction: PWS Maps, their Applications and Properties

Many real world processes in engineering, physics, biology, economics and other sciences, characterized by **'nonsmooth' phenomena** (such as sharp switching, impacts, friction, sliding and the like), are often modeled by means of **PWS functions** (Hommes, Nusse 1991, Day 1994, Matsuyama 1999, 2004, Zhusubaliyev, Mosekilde 2003, Gardini *et al.* 2008, di Bernardo *et al.* 2008, Bischi *et al.* 2009, etc.).

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PWS maps are characterized by

- existence of a **border (or switching manifold, or critical line)** across which the function changes its definition $\rightarrow$ **Border Collision Bifurcation (BCB)**, at which an invariant set collides with this border, and such a collision leads to a bifurcation (Nusse, Yorke 1992, 1995), e.g., a BCB of an attracting fixed point may lead directly to a chaotic attractor (di Bernardo *et al.* 1999, Gardini *et al.* 2010, Sushko, Gardini 2008);

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- **peculiar bifurcation structures** which are impossible in smooth systems, e.g., skew tent map bifurcation structure, period adding and period incrementing bifurcation structures, etc. (Avrutin, Schanz 2006, Sushko *et al.* 2015).

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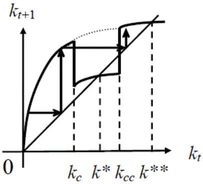
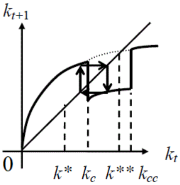
The model generates a rich array of dynamics (the variable is $k_t = K_t/L_t$ where K_t is physical capital, L_t is labor):



Credit Trap or Leapfrogging or Reversal of Fortune



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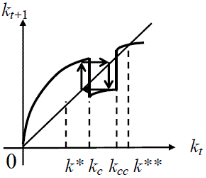

Cycles as a Trap or Growth Miracle

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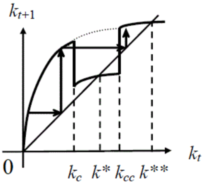

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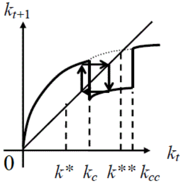

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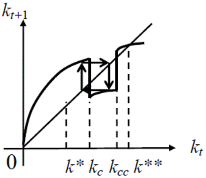

Cycles as a Trap or Growth Miracle

We offer a complete characterization of the dynamics for *Cobb-Douglas production function*, which makes the dynamical system **piecewise linear** (MSG, 2018).

Introduction: 1D discontinuous PWL maps

with one discontinuity point:

$$g : x \to g(x) = \begin{cases} g_L(x) = a_L x + \mu_L, & x < 0 \\ g_R(x) = a_R x + \mu_R, & x > 0 \end{cases}$$

$g_L(0) \neq g_R(0)$;

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$g_L(d_L) \neq g_M(d_L)$, $g_M(d_R) \neq g_R(d_R)$.

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Boundaries of periodicity regions in the parameter space

Suppose g has an *attracting* cycle of period $n \geq 1$. A boundary of the related periodicity region corresponds to either *border collision bifurcation* (BCB) of the cycle, or to a *degenerate bifurcation*.

Bifurcation structures in 1D discontinuous PWL maps

Period incrementing structure ($0 < a_L < 1$, $-1 < a_R < 0$)

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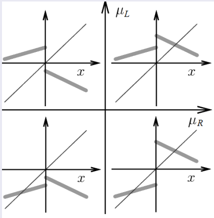

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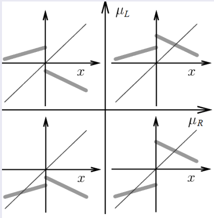

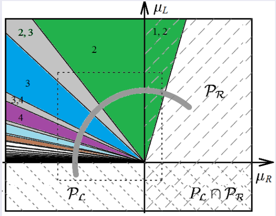

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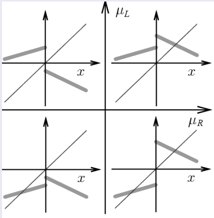

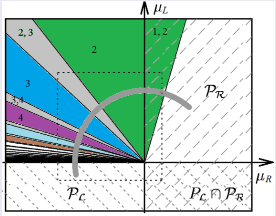

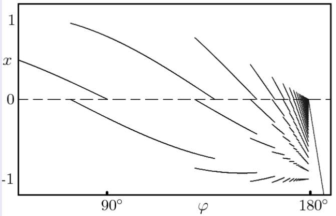

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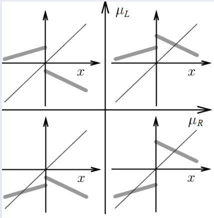

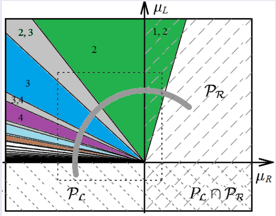

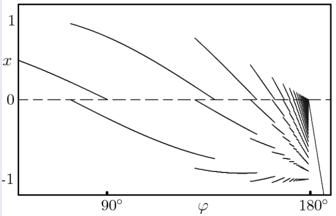

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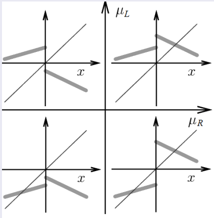

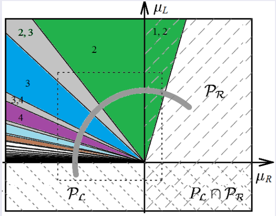

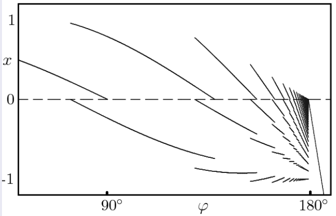

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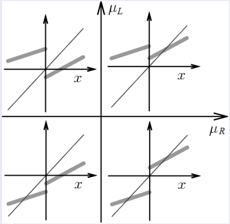

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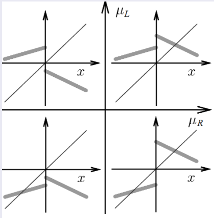

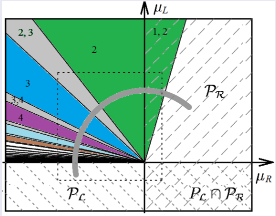

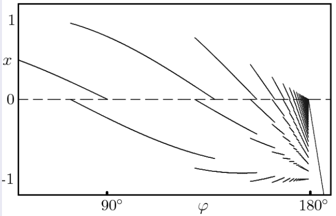

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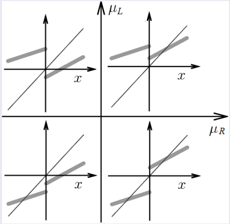

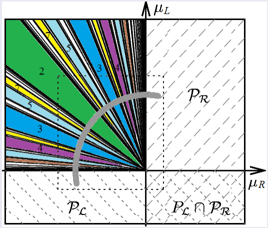

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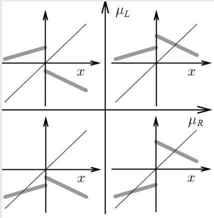

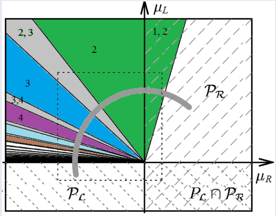

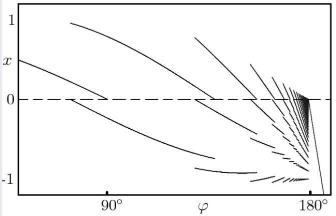

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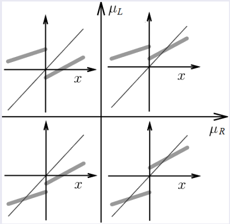

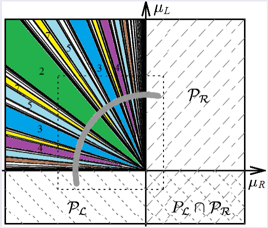

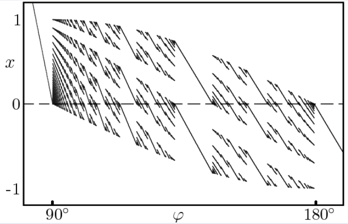

Credit cycle model with Cobb-Douglas production function

$f(k_t) = Ak^{\alpha}$, $0 < \alpha < 1$, after some variable and parameter transformations, is described by a 1D PWL map with two discontinuities:

$$g : x_{t+1} = g(x_t) = \begin{cases} g_L(x_t) = (1-\alpha) + \alpha x_t & \text{if } x_t < d_c \\ g_R(x_t) = \alpha x_t & \text{if } d_c < x_t < d_{cc} \\ g_U(x_t) = (1-\alpha) + \alpha x_t & \text{if } x_t > d_{cc} \end{cases}$$

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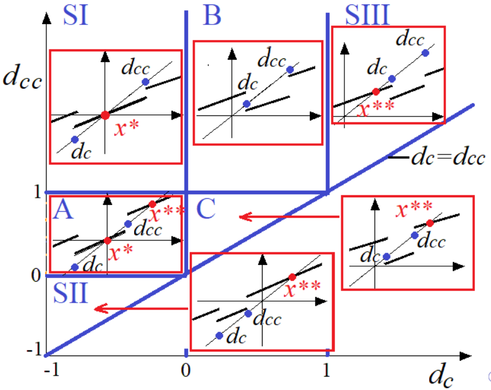
- Based on the existence of the fixed point, we can distinguish between the following parameter regions denoted A, B, C, SI, SII, SIII:



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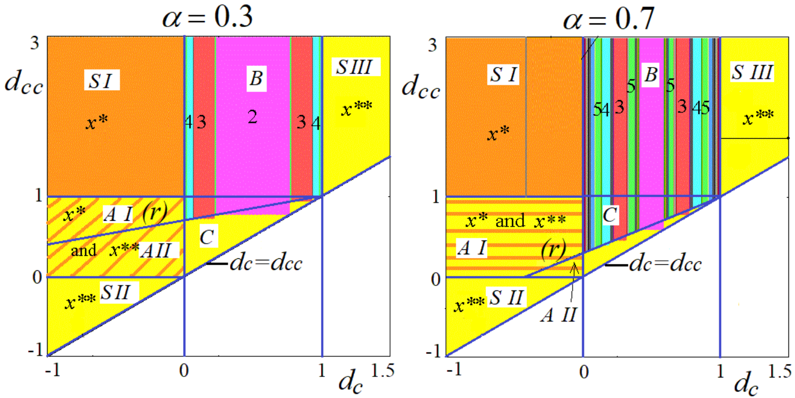
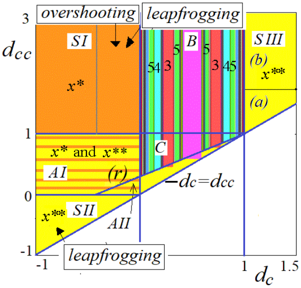


Cases SI, SII and SIII (globally attracting fixed points)



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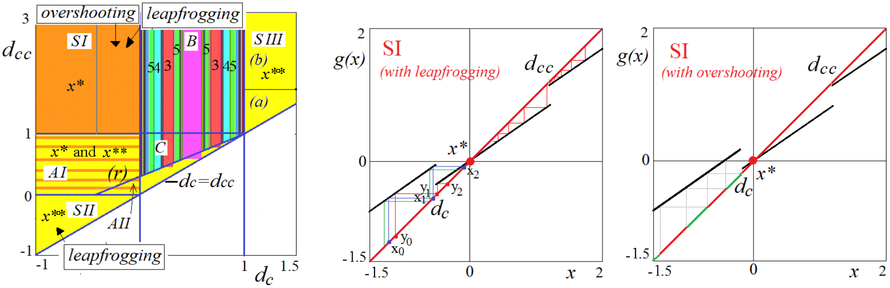

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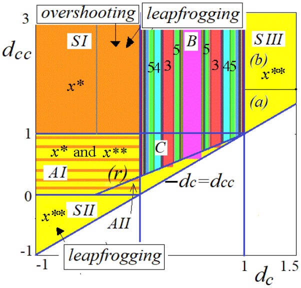

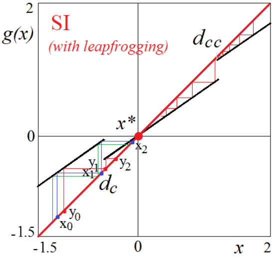

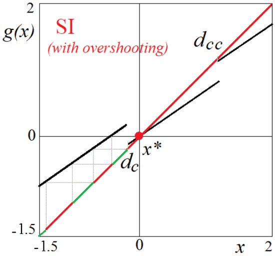

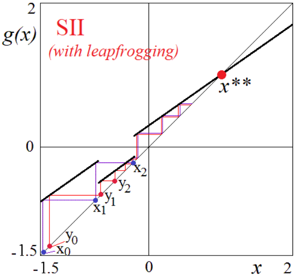

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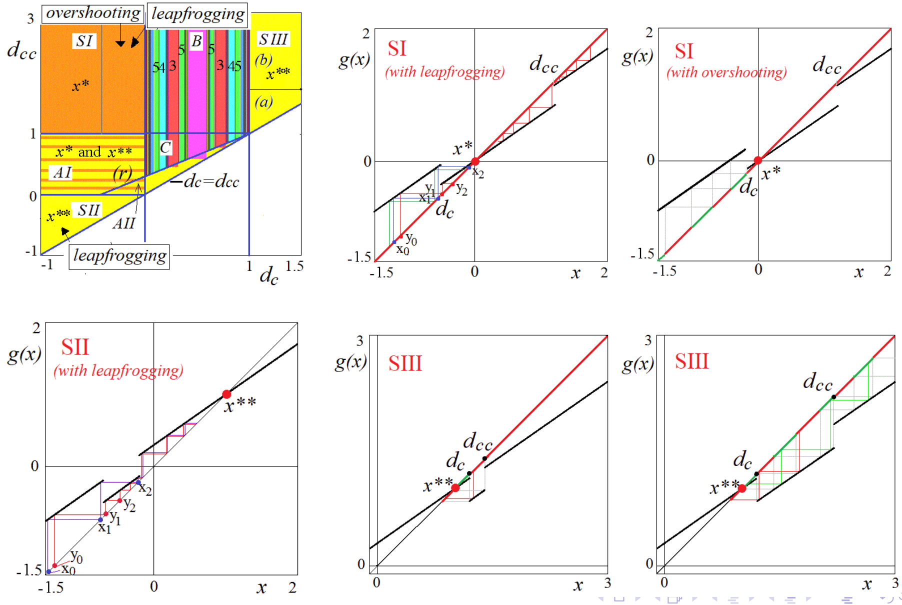
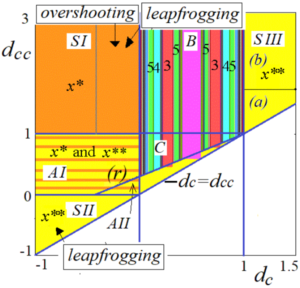


Cases AI, AII (coexisting attracting fixed points)



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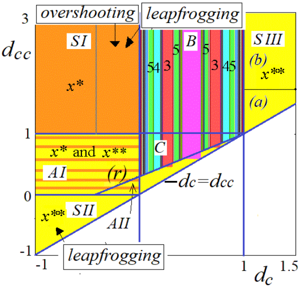

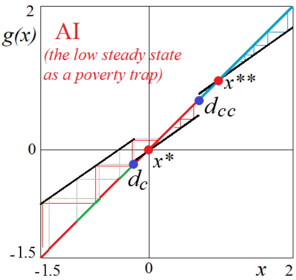

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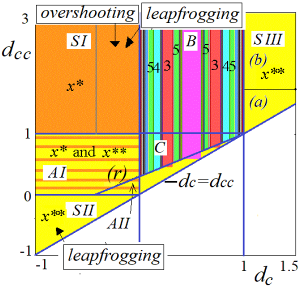

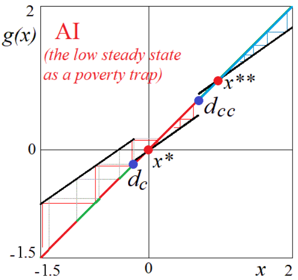

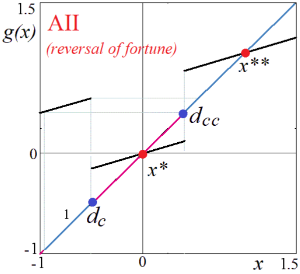

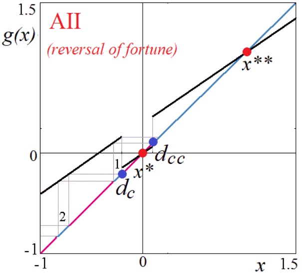

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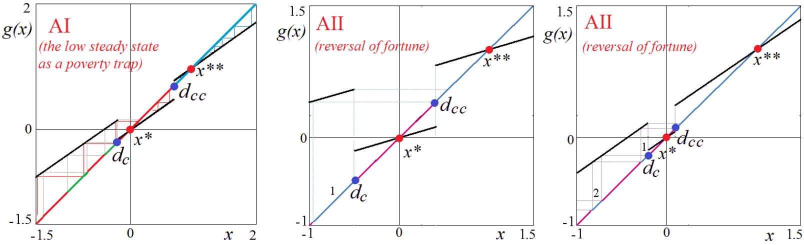
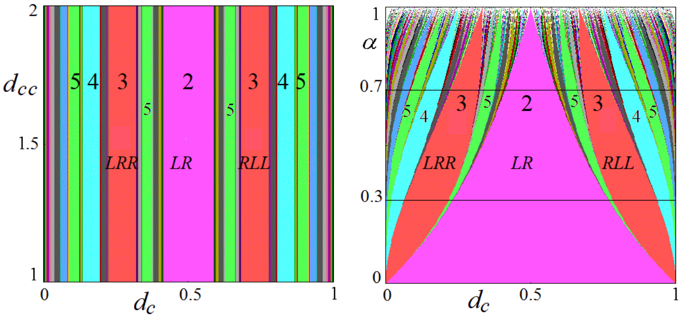


Proposition (Coexistence of attracting fixed points)

For any value of the parameter $\alpha \in (0,1)$, when the parameter point (d_c, d_{cc}) belongs to regions A-I or A-II then the attracting fixed points $x^* = 0$ and $x^{**} = 1$ coexist. Their basins of attraction for AI are connected and consist in two intervals, $\mathcal{B}(0) = (-\infty, d_{cc})$ and $\mathcal{B}(1) = (d_{cc}, +\infty)$, while for AII they are disconnected and formed by infinitely many alternating intervals, $\mathcal{B}(0) = (d_c, d_{cc}) \cup_{n>0} g_L^{-n}((d_c, d_{cc}))$ and $\mathcal{B}(1) = (d_{cc}, +\infty) \cup_{n>0} g_L^{-n}(J)$, where $J = (d_{cc}, g_L(d_c)]$.

Case B: n-cycles for any $n > 1$



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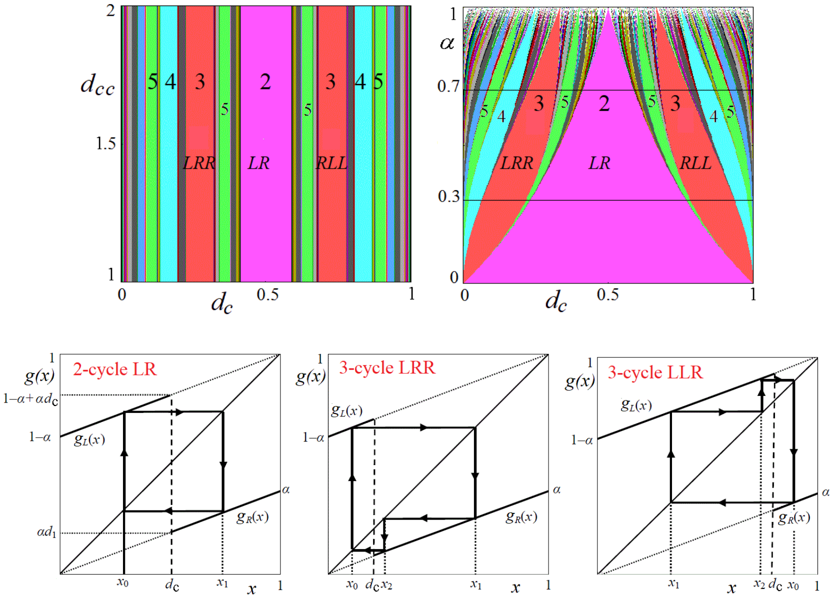

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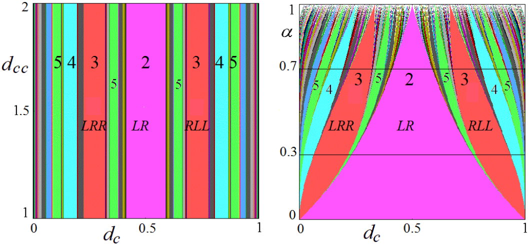

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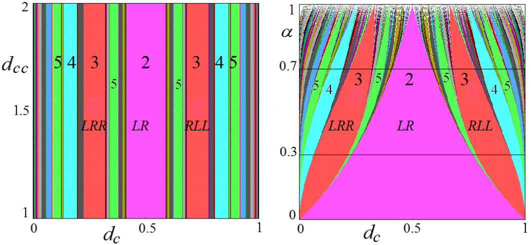

Proposition (period adding structure: first complexity level)

For any $\alpha \in (0, 1)$ and $(d_c, d_{cc}) \in B$, the cycle LR^n exists for

$$d_c \in \left(\frac{(1-\alpha)\alpha^n}{1-\alpha^{n+1}}, \frac{(1-\alpha)\alpha^{n-1}}{1-\alpha^{n+1}} \right) \quad (region\ \Pi_{LR^n})$$

while the cycle RL^n exists for

$$d_c \in \left(1 - \frac{(1-\alpha)\alpha^{n-1}}{1-\alpha^{n+1}}, 1 - \frac{(1-\alpha)\alpha^n}{1-\alpha^{n+1}} \right) \quad (region\ \Pi_{RL^n})$$

For any fixed $n > 1$, in the (d_c, d_{cc})-parameter plane the regions Π_{LR^n} and Π_{RL^n} are symmetric wrt the line $d_c = 0.5$; the region Π_{LR} is itself symmetric wrt $d_c = 0.5$.

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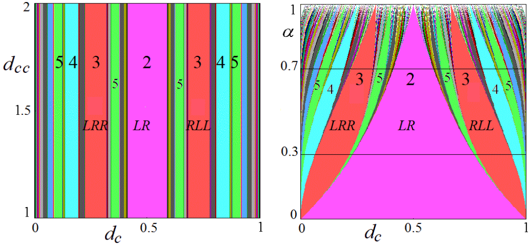

Higher complexity levels

Constructing proper **first return map**, one can show that there are two infinite sequences of periodicity regions of cycles of the **second complexity level**, $LR^n(LR^{n+1})^m$ and $(LR^n)^m LR^{n+1}$ for any integer $m \geq 1$, accumulating as $m \to \infty$ to $\Pi_{LR^{n+1}}$ and Π_{LR^n}, respectively.

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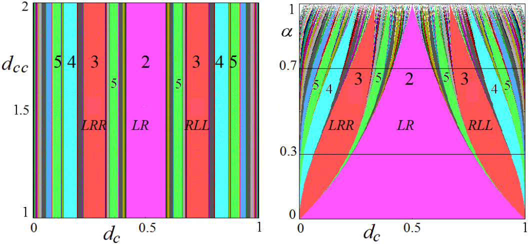

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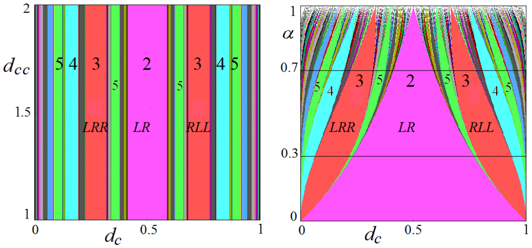

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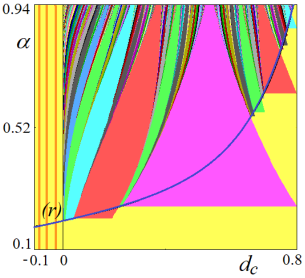
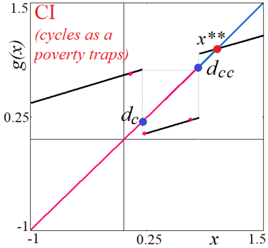
The union of all these regions does not cover the entire interval $d_c \in (0, 1)$. For the remaining set (of measure 0) the trajectory is *quasiperiodic*, dense in the invariant set, which is a Cantor set (see Hao 1989, Keener 1980, Avrutin et al. 2019).

Cases CI, CII: x^{**} coexisting with n-cycles $n > 1$





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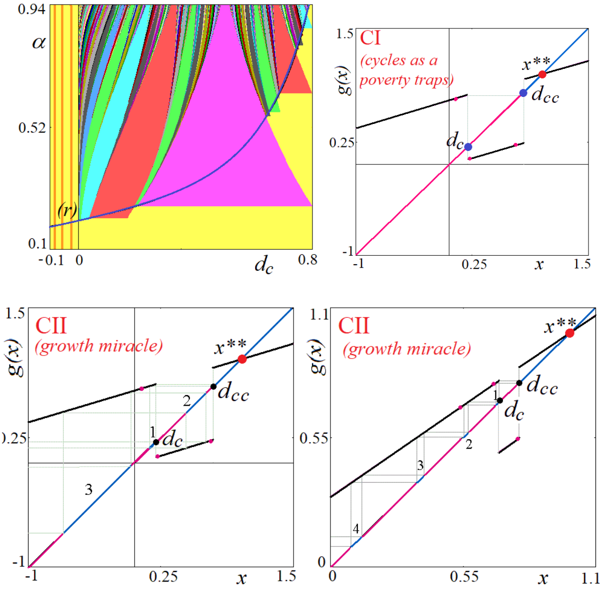

Conclusions

A **regime-switching model of credit frictions**, proposed by Matsuyama (2007a), can display a wide array of dynamical behavior. We propose a complete characterization of the dynamic behavior of this model for the **Cobb-Douglas case**, which makes the dynamical system piecewise linear.

Conclusions

A **regime-switching model of credit frictions**, proposed by Matsuyama (2007a), can display a wide array of dynamical behavior. We propose a complete characterization of the dynamic behavior of this model for the **Cobb-Douglas case**, which makes the dynamical system piecewise linear. Among others, we show

- How overshooting, leapfrogging and reversal of fortune can occur.
- How stable cycles of any period can emerge.
- Along each stable cycle, how the economy alternates between the expansionary and contractionary phases.
- How asymmetry of cycles (the fraction of time the economy is in the expansionary phase) varies with the credit frictions parameters.
- How the economy may fluctuate for a long time at a lower level before successfully escaping from the poverty, etc.

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The analysis was done for a restrictive set of assumptions, with only two projects and two switching points, because it is sufficient to create a rich array of dynamics with a relatively few parameters. Obviously, with more projects and more switching points, the model would generate even richer behaviors.

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What simplifies the analysis is the discontinuity and piecewise linearity of the dynamics. Similar results can be numerically obtained with a piecewise smooth discontinuous map and also when the discontinuous piecewise linear or piecewise smooth map is approximated by a continuous map with very steep slopes.

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