Heterogeneous expectations, housing bubbles and tax policy

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11th Conference on Nonlinear Economic Dynamics
September 4-6, Kyiv, Ukraine

Starting point and goal

- History is replete with dramatic housing market bubbles that had serious effects for the real economy.
- We therefore seek to develop a plausible housing market model that helps us to understand such dynamics.
- We also explore to which extent policy makers can stabilize housing markets by adjusting housing market related taxes.

Related literature

- Poterba's (1984, 1991) housing market model, including a rental market, a housing capital market and perfect foresight, reveals that housing market related taxes affect the user cost of housing. His main attention rests on the model's steady state and not on its dynamics.
- Dieci and Westerhoff (2016) add heterogeneous expectations (switching depends on market circumstances) to Poterba's model and show that nonlinear interactions between real and speculative forces may lead to complex housing market dynamics.

Related literature

- In our paper, we combine the housing market model by Poterba (1984, 1991) with the heuristic switching approach by Brock and Hommes (1997, 1998). Our model produces endogenous boom-bust dynamics, which can be tamed via housing market related taxes.
- Currently, we see a boom in this line of research: Bolt et al. (2019), Burnside et al. (2016), Campisi et al. (2018), Dieci and Westerhoff (2012), Diks and Wang (2016), Eichholtz et al. (2015), Glaeser and Nathanson (2017), Kouwenberg and Zwinkels (2014), Schmitt and Westerhoff (2019), ...

A behavioral housing market model

- According to Poterba's (1984, 1991) **rental market** setup, the **market clearing condition for housing services** is defined as
  (1) $D_t = S_t$
- The **demand for** and the **supply of housing services** is described as
  (2) $D_t = a - bR_t$ and (3) $S_t = cH_t$
- By inserting (2) and (3) in (1), the **rent level** R_t is given by
  (4) $R_t = \alpha - \beta H_t$, where $\alpha = \frac{a}{b} > 0$ and $\beta = \frac{c}{b} > 0$

A behavioral housing market model

- Poterba's (1984, 1991) **housing capital market** setup entails a **market clearing condition for housing stock**

  (5) $Z_t = H_t$

- The **development of the housing stock** is given by

  (6) $H_t = I_t + (1 - \delta)H_{t-1}$

- Since **housing investments** in period t are described as

  (7) $I_t = \gamma P_{t-1}$,

  we obtain the **evolution of the housing stock** as

  (8) $H_t = \gamma P_{t-1} + (1 - \delta)H_{t-1}$

A behavioral housing market model

- For a hypothetical house price level P_t at time t, **investor i's end-of-period wealth** is formalized as

  (9) $W_{t+1}^i = (1+r)W_t^i + Z_t^i(P_{t+1} + R_t - (1+r+\delta)P_t) - c^i$

- Investor i's **mean-variance optimization problem** is modeled by

  (10) $max_{Z_t^i} \left\{ E_t^i[W_{t+1}^i] - \frac{\lambda^i}{2} V_t^i[W_{t+1}^i] \right\}$

- Investor i's **solution to the above maximization problem** yields

  (11) $Z_t^i = \frac{E_t^i[P_{t+1}] + R_t - (1+r+\delta)P_t}{\lambda^i V_t^i[P_{t+1}]}$

A behavioral housing market model

- Introducing some "standard" assumptions allows us to express **investors' total housing demand** as

  (12) $Z_t = \frac{E_t[P_{t+1}]+R_t-(1+r+\delta)P_t}{\lambda\sigma^2}$

- From the market clearing condition for housing stock (5), it then follows that

  (13) $P_t = \frac{E_t[P_{t+1}]+R_t-H_t\lambda\sigma^2}{1+r+\delta}$

A behavioral housing market model

- **Extrapolative and regressive expectations** are given by

  (14) $E_t^E[P_{t+1}] = P_{t-1} + \chi(P_{t-1} - P_{t-2})$

  (15) $E_t^R[P_{t+1}] = P_{t-1} + \phi(\overline{P} - P_{t-1})$

- The **market shares of extrapolators and fundamentalists** are formalized as

  (16) $N_t^E = \frac{exp[\nu A_t^E]}{exp[\nu A_t^E]+exp[\nu A_t^R]}$

  (17) $N_t^R = \frac{exp[\nu A_t^R]}{exp[\nu A_t^E]+exp[\nu A_t^R]}$

A behavioral housing market model

- The **fitness of extrapolative and regressive expectations** is modeled by

  (18) $A_t^E = (P_{t-1} + R_{t-2} - (1 + r + \delta)P_{t-2})Z_{t-2}^E$

  (19) $A_t^R = (P_{t-1} + R_{t-2} - (1 + r + \delta)P_{t-2})Z_{t-2}^R - c$

- Investors' **average house price expectations** are defined by

  (20) $E_t[P_{t+1}] = N_t^E E_t^E[P_{t+1}] + N_t^R E_t^R[P_{t+1}]$

Stability analysis

Proposition 1

The model's unique steady state, implying, amongst others, that $\overline{P} = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma}$ and $\overline{H} = \frac{\alpha\gamma}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma}$, is locally asymptotically stable if and only if

(i) $\overline{N}^E\chi\delta + \frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$

and

(ii) $\overline{N}^R\phi + \frac{\gamma(\beta+\lambda\sigma^2)}{2-\delta} < 2 + r + \delta + 2\overline{N}^E\chi,$

where $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ and $\overline{N}^R = \frac{1}{1+exp[\nu c]}$, respectively.

Moreover, violation of the first (second) inequality is associated with a Neimark-Sacker (Flip) bifurcation.

Interpretation of FSS

- The model's unique fundamental steady state implies that
$\overline{P} = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma}$, $\overline{H} = \frac{\alpha\gamma}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma}$, $\overline{R} = \alpha - \beta\overline{H}$,
$\overline{N}^E = \frac{1}{1+exp[-\nu c]}$, $\overline{N}^R = \frac{1}{1+exp[\nu c]}$
- Effects of real parameters:
  - $r\uparrow$: $\overline{P}\downarrow$, $\overline{H}\downarrow$, $\overline{R}\uparrow$
  - $\gamma\uparrow$: $\overline{P}\downarrow$, $\overline{H}\uparrow$, $\overline{R}\downarrow$
- Effects of behavioral parameters:
  - $c\uparrow$: $\overline{N^E}\uparrow$, $\overline{N^R}\downarrow$
  - $\chi\uparrow$: no effects

Interpretation of NS condition

- Effects of behavioral parameters:
  - $\chi = 0$, $0 < \phi < 1$: FSS is always stable
  - $\chi \uparrow$: stability may be lost
  - $\phi \uparrow$: stability may be regained
  - c or $\nu \uparrow$: stability may be lost (via $\overline{N^E} \uparrow$)
- Effects of real parameters:
  - $r \uparrow$: beneficial for stability
  - $\gamma \uparrow$: harmful for stability

Base parameter setting

- Our base parameter setting implies that $\overline{P} = 1$, $\overline{H} = 20$, $\overline{R} = 0.3$ and $\overline{N^E} = 0.731$
- Steady state is unstable due to a Neimark-Sacker bifurcation, e.g. $\chi = 1.1$ exceeds $\chi^{NS}_{crit} = 1.08$
- One time step corresponds (roughly) to one year
- Next: time series examples and bifurcation diagrams

Dynamics for base parameter setting

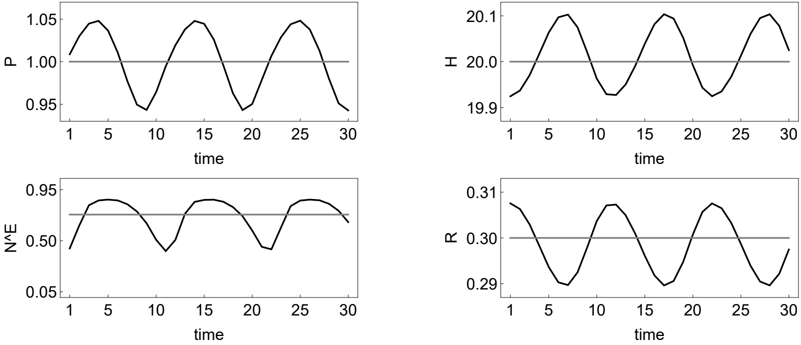

Figure 1: Model dynamics for base parameter setting.

Dynamics for alternative parameter setting

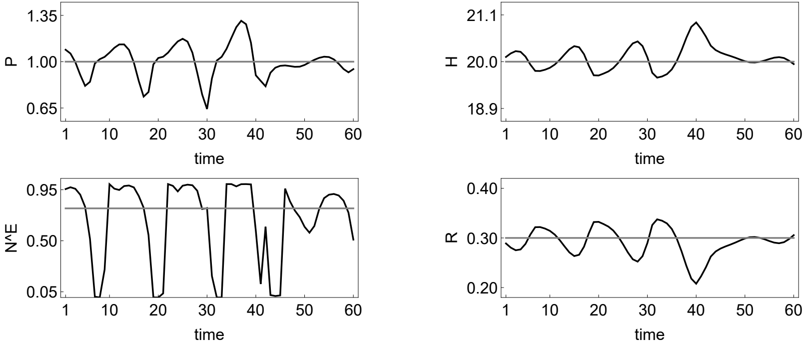

Figure 2: Model dynamics for $\chi = 1.35$, $\phi = 0.75$, $\nu = 1.3$.

Effects of behavioral parameters

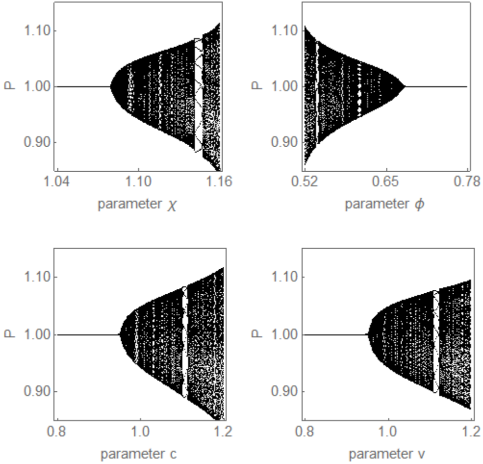

Figure 3: Effects of behavioral parameters.

Effects of real parameters

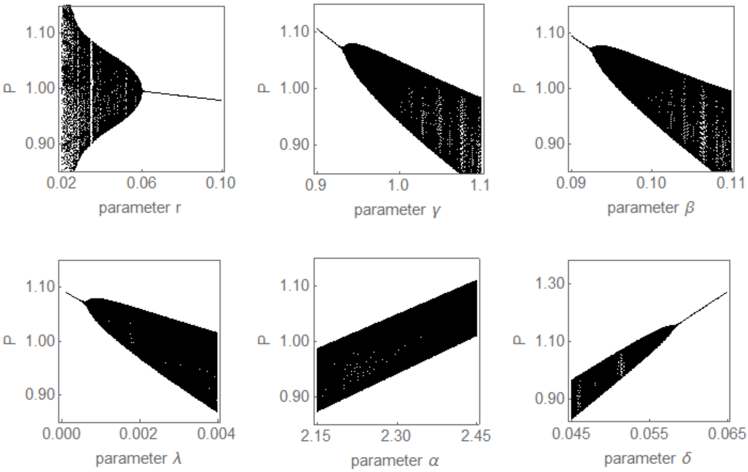

Figure 4: Effects of real parameters.

Tax policies

We consider the following tax policies:

(1) Tax on purchase of houses

(2) Tax on rental income

(3) Tax on owning housing stock

(4) Revenue tax for housing constructors

(5) Tax on wealth of investors

(6) Tax deductibility of information cost

Our focus is on the model's fundamental steady state and on the Neimark-Sacker stability condition, summarized by Propositions 2-7 and illustrated via bifurcation diagrams.

Policy 1: Tax on purchase of houses

- A tax on house purchases affects investor i's wealth equation:
$W^i_{t+1} = (1+r)W^i_t + Z^i_t(P_{t+1} + R_t - (1+r+\delta+\tau)P_t) - c^i$

Proposition 2

The model's new unique steady state is given by $\overline{P} = \frac{\alpha\delta}{(r+\delta+\tau)\delta+(\beta+\lambda\sigma^2)\gamma}$ and $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ and implies that $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ and $\overline{N}^R = \frac{1}{1+exp[\nu c]}$.
If $\overline{N}^E\chi\delta + \frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta+\tau-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r+\tau}{1-\delta}$ is violated, the steady state undergoes a Neimark-Sacker bifurcation and becomes unstable.

- Hence: $\tau\uparrow$: $\overline{P}\downarrow$, $\overline{H}\downarrow$, $\overline{R}\uparrow$, stability domain increases

Effects of taxes

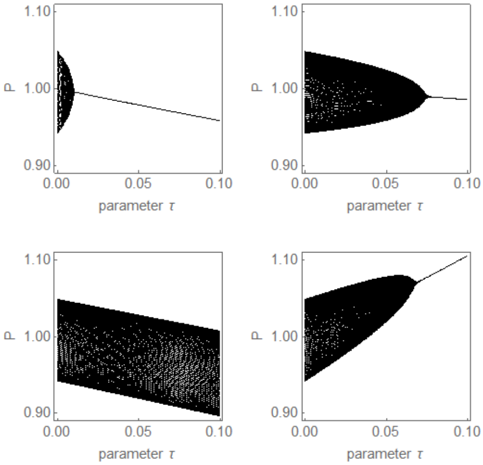

Figure 5: Effects of taxes on purchases of houses, rental income, owning housing stock and revenue of housing constructors, respectively.

Policy 2: Tax on rental income

- A tax on rental income affects investor i's wealth equation: $W^i_{t+1} = (1+r)W^i_t + Z^i_t(P_{t+1} + (1-\tau)R_t - (1+r+\delta)P_t) - c^i$

Proposition 3

The model's new unique steady state is given by $\overline{P} = \frac{(1-\tau)\alpha\delta}{(r+\delta)\delta+((1-\tau)\beta+\lambda\sigma^2)\gamma}$ and $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ and implies that $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ and $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. If $\overline{N}^E\chi\delta + \frac{\gamma((1-\tau)\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ is violated, the steady state undergoes a Neimark-Sacker bifurcation and becomes unstable.

- Hence: $\tau\uparrow$: $\overline{P}\downarrow$, $\overline{H}\downarrow$, $\overline{R}\uparrow$, stability domain increases

Policy 3: Tax on owning housing stock

- A tax on housing stock affects investor i's wealth equation:
$W_{t+1}^i = (1+r)W_t^i + Z_t^i(P_{t+1} + R_t - \tau - (1+r+\delta)P_t) - c^i$

Proposition 4

The model's new unique steady state is given by $\overline{P} = \frac{(\alpha-\tau)\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)\gamma}$ and $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ and implies that $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ and $\overline{N}^R = \frac{1}{1+exp[\nu c]}$.
If $\overline{N}^E\chi\delta + \frac{\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ is violated, the steady state undergoes a Neimark-Sacker bifurcation and becomes unstable.

- Hence: $\tau\uparrow$: $\overline{P}\downarrow$, $\overline{H}\downarrow$, $\overline{R}\uparrow$, no effect on stability domain

Policy 4: Revenue tax for housing constructors

- A revenue tax for housing constructors affects their profit maximization problem:
  $max_{I_t}\{(1-\tau)E_{t-1}[P_t]I_t - C_t\}$

Proposition 5

The model's new unique steady state is given by
$\overline{P} = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+\lambda\sigma^2)(1-\tau)\gamma}$ and $\overline{H} = \frac{(1-\tau)\gamma}{\delta}\overline{P}$ and implies that
$\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ and $\overline{N}^R = \frac{1}{1+exp[\nu c]}$.

If $\overline{N}^E\chi\delta + \frac{(1-\tau)\gamma(\beta+\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ is violated, the steady state undergoes a Neimark-Sacker bifurcation and becomes unstable.

- Hence: $\tau\uparrow$: $\overline{P}\uparrow$, $\overline{H}\downarrow$, $\overline{R}\uparrow$, stability domain increases

Policy 5: Tax on wealth of investors

- A wealth tax affects investor i's wealth equation:
$W^i_{t+1} = (1-\tau)((1+r)W^i_t + Z^i_t(P_{t+1} + R_t - (1+r+\delta)P_t)) - c^i$

Proposition 6

The model's new unique steady state is given by
$\overline{P} = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+(1-\tau)\lambda\sigma^2)\gamma}$ and $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ and implies that $\overline{N}^E = \frac{1}{1+exp[-\nu c]}$ and $\overline{N}^R = \frac{1}{1+exp[\nu c]}$. If $\overline{N}^E\chi\delta + \frac{\gamma(\beta+(1-\tau)\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ is violated, the steady state undergoes a Neimark-Sacker bifurcation and becomes unstable.

- Hence: $\tau\uparrow$: $\overline{P}\uparrow$, $\overline{H}\uparrow$, $\overline{R}\downarrow$, stability domain increases

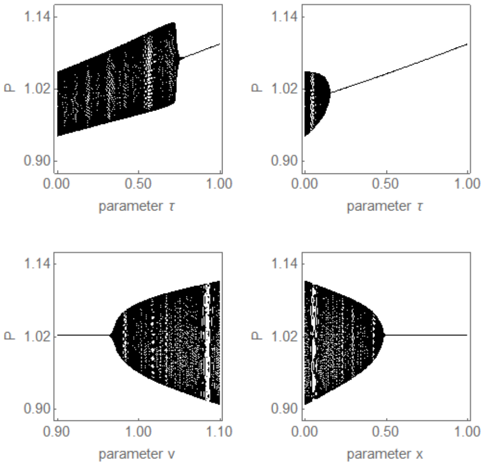

Figure 6: Effects of wealth taxes and partial deductibility of information costs.

Policy 6: Deductibility of information cost

- Deductibility of information cost, wealth of fundamentalists: $W^R_{t+1} = (1-\tau)((1+r)W_t + Z_t(P_{t+1} + R_t - (1+r+\delta)P_t)) - c(1-\tau x)$, where x denotes fraction of deductibility

Proposition 7

The model's new unique steady state is given by $\overline{P} = \frac{\alpha\delta}{(r+\delta)\delta+(\beta+(1-\tau)\lambda\sigma^2)\gamma}$ and $\overline{H} = \frac{\gamma}{\delta}\overline{P}$ and implies that $\overline{N}^E = \frac{1}{1+exp[-(1-\tau x)\nu c]}$ and $\overline{N}^R = \frac{1}{1+exp[(1-\tau x)\nu c]}$.

If $\overline{N}^E\chi\delta + \frac{\gamma(\beta+(1-\tau)\lambda\sigma^2)\overline{N}^E\chi}{1+r+\delta-\overline{N}^E\chi} < \overline{N}^R\phi + \frac{2\delta+r}{1-\delta}$ is violated, the steady state undergoes a Neimark-Sacker bifurcation and becomes unstable.

- Hence: $x\uparrow$: $\overline{P}$, $\overline{H}$, $\overline{R}$ remain constant, stability domain increases via $\overline{N}^E \downarrow$

Conclusions

- We develop a novel housing market model that rests on Poterba's (1984, 1991) user cost framework and Brock and Hommes' (1997, 1998) heuristic switching approach.
- Our model generates endogenous boom-bust dynamics, provided that investors extrapolate past price changes sufficiently strongly. However, the housing market's real side also matters.
- Our model allows to investigate the effects of housing market related taxes on the model's steady state, stability and out-of-equilibrium behavior.