Disposition Effect in a financial market model with heterogeneous agents

Alessia Cafferata[1] and Fabio Tramontana[2]

[1]Department of Economics,
University of Genova (Italy) ,
[2]Department of Mathematical Sciences, Mathematical Finance and Econometrics
Catholic University of Milan (Italy)

11th NED Conference 2019 - Kyiv - Sept 4-6, 2019

Outline

Cognitive limitations

- It is nowadays well established that *people rely on simple heuristics for their decision making* (since Kahneman and Tversky's pioneering works);
- Consumers, firms, investors, governments, ecc... are all affected by *biases* that lead to *systematic deviations* from a perfectly rational behavior;
- In parallel with an experimental literature on cognitive limitations, *a theoretical one has been developed* to replace the neoclassical models and explore that consequences of more realistic behavioral assumptions (the most famous models are probably *Prospect Theory, Hyperbolic discounting and Non-bayesian updating*);
- One of the most interesting fields of applications is Finance, and in particular *the behavior of investors*.

Cognitive limitations

- It is nowadays well established that *people rely on simple heuristics for their decision making* (since Kahneman and Tversky's pioneering works);
- Consumers, firms, investors, governments, ecc... are all affected by *biases* that lead to *systematic deviations* from a perfectly rational behavior;
- In parallel with an experimental literature on cognitive limitations, *a theoretical one has been developed* to replace the neoclassical models and explore that consequences of more realistic behavioral assumptions (the most famous models are probably *Prospect Theory, Hyperbolic discounting and Non-bayesian updating*);
- One of the most interesting fields of applications is Finance, and in particular *the behavior of investors*.

Cognitive limitations

- It is nowadays well established that *people rely on simple heuristics for their decision making* (since Kahneman and Tversky's pioneering works);
- Consumers, firms, investors, governments, ecc... are all affected by *biases* that lead to *systematic deviations* from a perfectly rational behavior;
- In parallel with an experimental literature on cognitive limitations, *a theoretical one has been developed* to replace the neoclassical models and explore that consequences of more realistic behavioral assumptions (the most famous models are probably *Prospect Theory, Hyperbolic discounting and Non-bayesian updating*);
- One of the most interesting fields of applications is Finance, and in particular *the behavior of investors*.

Cognitive limitations

- It is nowadays well established that *people rely on simple heuristics for their decision making* (since Kahneman and Tversky's pioneering works);
- Consumers, firms, investors, governments, ecc... are all affected by *biases* that lead to *systematic deviations* from a perfectly rational behavior;
- In parallel with an experimental literature on cognitive limitations, *a theoretical one has been developed* to replace the neoclassical models and explore that consequences of more realistic behavioral assumptions (the most famous models are probably *Prospect Theory, Hyperbolic discounting and Non-bayesian updating*);
- One of the most interesting fields of applications is Finance, and in particular *the behavior of investors*.

The Disposition Effect

- One of the most studied behavioral anomalies affecting investors is their *"tendency to sell winners too early and ride losers too long"*
- Shefrin and Statman (1985) refer to this tendency as *Disposition Effect (DE)*, and this label has been commonly accepted by researchers;
- This behavior, inconsistent with the assumption of perfect rationality of investors, is usually considered a consequence of other biases such as: *mental acconting, regret aversion, self-control problems, prospect theory preferences*.

The Disposition Effect

- One of the most studied behavioral anomalies affecting investors is their *"tendency to sell winners too early and ride losers too long"*
- Shefrin and Statman (1985) refer to this tendency as *Disposition Effect (DE)*, and this label has been commonly accepted by researchers;
- This behavior, inconsistent with the assumption of perfect rationality of investors, is usually considered a consequence of other biases such as: *mental acconting, regret aversion, self-control problems, prospect theory preferences.*

The Disposition Effect

- One of the most studied behavioral anomalies affecting investors is their *"tendency to sell winners too early and ride losers too long"*
- Shefrin and Statman (1985) refer to this tendency as *Disposition Effect (DE)*, and this label has been commonly accepted by researchers;
- This behavior, inconsistent with the assumption of perfect rationality of investors, is usually considered a consequence of other biases such as: *mental acconting, regret aversion, self-control problems, prospect theory preferences*.

Modelization and Consequences of the DE

- It is now deeply studied the effect of the DE on the *profitability of momentum strategies* ;
- Among the theoretical works dealing with the DE, the one of *Grinblatt and Han (2005)* appers to be particularly relevant. They also were interested in the link between DE and Momentum strategies.
- The DE is still the object of a huge amount of the literature (see among the most recent works Meng & Weng (2019), Li and Yang (2013), Barberis & Xiong (2009));

Modelization and Consequences of the DE

- It is now deeply studied the effect of the DE on the *profitability of momentum strategies* ;
- Among the theoretical works dealing with the DE, the one of *Grinblatt and Han (2005)* appers to be particularly relevant. They also were interested in the link between DE and Momentum strategies.
- The DE is still the object of a huge amount of the literature (see among the most recent works Meng & Weng (2019), Li and Yang (2013), Barberis & Xiong (2009));

Modelization and Consequences of the DE

- It is now deeply studied the effect of the DE on the *profitability of momentum strategies* ;
- Among the theoretical works dealing with the DE, the one of *Grinblatt and Han (2005)* appers to be particularly relevant. They also were interested in the link between DE and Momentum strategies.
- The DE is still the object of a huge amount of the literature (see among the most recent works Meng & Weng (2019), Li and Yang (2013), Barberis & Xiong (2009));

Trading responses to past returns

- *Ben-David e Hirshleifer (2012)* analyzed a set of US investors finding that...

"The probability of either selling or buying has a [asymmetric] V-shaped relation to profits, with a minimum at zero"

Trading responses to past returns

- *Ben-David e Hirshleifer (2012)* analyzed a set of US investors finding that...

"The probability of either selling or buying has a [asymmetric] V-shaped relation to profits, with a minimum at zero"

Trading responses to past returns

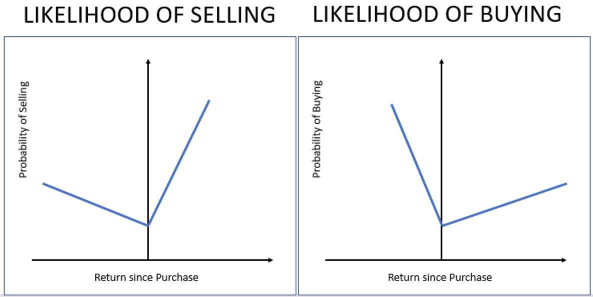
- *Ben-David e Hirshleifer (2012)* analyzed a set of US investors finding that...

"The probability of either selling or buying has a [asymmetric] V-shaped relation to profits, with a minimum at zero"

Trading responses to past returns

- *Ben-David e Hirshleifer (2012)* analyzed a set of US investors finding that...

"The probability of either selling or buying has a [asymmetric] V-shaped relation to profits, with a minimum at zero"



An HAM with 3 kinds of agents

We consider a market of a single asset, whose price (P) is regulated by *a market maker* as follows:

$$P' = P + \alpha \left[D^{f1} + D^{f2} + D^{c} \right]$$

with $\alpha > 0$ the *reactivity* of the market maker and D^{f1}, D^{f2} and D^{c} the excess demands of *fundamentalists (kind 1 and 2) and chartists*, respectively.

The excess demand of fundamentalists is *cubic*:

$$D^{f1} = f(F - P) \text{ and } D^{f2} = f(F - P)^3$$

where $f > 0$ is their *speed of reaction* and F is the fundamental value.

An HAM with 3 kinds of agents

We consider a market of a single asset, whose price (P) is regulated by *a market maker* as follows:

$$P' = P + \alpha \left[D^{f1} + D^{f2} + D^{c} \right]$$

with $\alpha > 0$ the *reactivity* of the market maker and D^{f1}, D^{f2} and D^{c} the excess demands of *fundamentalists (kind 1 and 2) and chartists*, respectively.
The excess demand of fundamentalists is *cubic*:

$$D^{f1} = f(F-P) \text{ and } D^{f2} = f(F-P)^{3}$$

where $f > 0$ is their *speed of reaction* and F is the fundamental value.

Modelling the DE

- We introduce DE only for chartists;
- We consider two different decisions: *buy or sell? For which amount?*
- They buy the asset when it is overvalued and sell it when it is undevalued
- Without DE the excess demand of chartists would be the following:

$$D^c = c(P - F)$$

where $c > 0$ is their *speed of reaction* and regultes the amount of buying/selling.

Modelling the DE

- We introduce DE only for chartists;
- We consider two different decisions: *buy or sell? For which amount?*
- They buy the asset when it is overvalued and sell it when it is undevalued
- Without DE the excess demand of chartists would be the following:

$$D^c = c(P - F)$$

where $c > 0$ is their *speed of reaction* and regultes the amount of buying/selling.

Modelling the DE

- We introduce DE only for chartists;
- We consider two different decisions: *buy or sell? For which amount?*
- They buy the asset when it is overvalued and sell it when it is undevalued
- Without DE the excess demand of chartists would be the following:

$$D^c = c(P - F)$$

where $c > 0$ is their *speed of reaction* and regultes the amount of buying/selling.

Modelling the DE

- We introduce DE only for chartists;
- We consider two different decisions: *buy or sell? For which amount?*
- They buy the asset when it is overvalued and sell it when it is undevalued
- Without DE the excess demand of chartists would be the following:

$$D^c = c(P - F)$$

where $c > 0$ is their *speed of reaction* and regultes the amount of buying/selling.

Modelling the DE

- We introduce DE only for chartists;
- We consider two different decisions: *buy or sell? For which amount?*
- They buy the asset when it is overvalued and sell it when it is undevalued
- Without DE the excess demand of chartists would be the following:

$$D^c = c(P - F)$$

where $c > 0$ is their *speed of reaction* and regultes the amount of buying/selling.

Modelling the DE

- We *endogenize the speed of reaction* c in order to make the behavior of chartists consistent with the findings of Ben-David and Hirshleifer (asymmetric V-shaped schedules);
- In order to introduce losses or gains *we need a reference point*. We follow Grinblatt and Han (2005) using *a weighted average of past prices as reference price* $\widetilde{P}$:

$$c(P,\widetilde{P}) \equiv \begin{cases} c + s_g(P - \widetilde{P}) & \text{if} \quad P < F \cup P > \widetilde{P} \text{ (i)} \\ c - s_l(P - \widetilde{P}) & \text{if} \quad P < F \cup P < \widetilde{P} \text{ (ii)} \\ c + b_g(P - \widetilde{P}) & \text{if} \quad P > F \cup P > \widetilde{P} \text{(iii)} \\ c - b_l(P - \widetilde{P}) & \text{if} \quad P > F \cup P < \widetilde{P} \text{ (iv)} \end{cases}$$

where $s_g \geq s_l \geq 0$ and $b_l \geq b_g \geq 0$.

Modelling the DE

- We *endogenize the speed of reaction* c in order to make the behavior of chartists consistent with the findings of Ben-David and Hirshleifer (asymmetric V-shaped schedules);
- In order to introduce losses or gains *we need a reference point*. We follow Grinblatt and Han (2005) using *a weighted average of past prices as reference price* $\widetilde{P}$:

$$c(P,\widetilde{P}) \equiv \begin{cases} c + s_g(P - \widetilde{P}) & \text{if} \quad P < F \cup P > \widetilde{P} \text{ (i)} \\ c - s_l(P - \widetilde{P}) & \text{if} \quad P < F \cup P < \widetilde{P} \text{ (ii)} \\ c + b_g(P - \widetilde{P}) & \text{if} \quad P > F \cup P > \widetilde{P} \text{(iii)} \\ c - b_l(P - \widetilde{P}) & \text{if} \quad P > F \cup P < \widetilde{P} \text{ (iv)} \end{cases}$$

where $s_g \geq s_l \geq 0$ and $b_l \geq b_g \geq 0$.

The Dynamic model

- By using these definitions into the dynamic equation of the market maker we finally get:

$$T = \begin{cases} P' = P + \alpha \left[f(F-P) + f(F-P)^3 + c(P, \widetilde{P})(P-F) \right] \\ \widetilde{P}' = \lambda \widetilde{P} + (1-\lambda) P \end{cases}$$

where $0 \leq \lambda \leq 1$ measures the weight of the last reference price in the formation of the new one.

- This a two-dimensional piecewise defined dynamical system.

The Dynamic model

- By using these definitions into the dynamic equation of the market maker we finally get:

$$T = \begin{cases} P' = P + \alpha \left[f(F-P) + f(F-P)^3 + c(P, \widetilde{P})(P-F) \right] \\ \widetilde{P}' = \lambda \widetilde{P} + (1-\lambda) P \end{cases}$$

where $0 \leq \lambda \leq 1$ measures the weight of the last reference price in the formation of the new one.

- This a two-dimensional piecewise defined dynamical system.

Equilibria

By using the equilibrium conditions $P' = P = P^*$ and $\widetilde{P}' = \widetilde{P} = \widetilde{P}^*$ it is immediate to obtain that in equilibrium:

$$P^* = \widetilde{P}^*$$

so *the reference price is equal to the equilibrium price*.
For strictly positive values of the parameters three equilibria always exist:

$$P^*_F = F; \quad P^*_+ = F + \sqrt{\frac{c}{f} - 1}; \quad P^*_- = F - \sqrt{\frac{c}{f} - 1}$$

Equilibria

By using the equilibrium conditions $P' = P = P^*$ and $\widetilde{P}' = \widetilde{P} = \widetilde{P}^*$ it is immediate to obtain that in equilibrium:

$$P^* = \widetilde{P}^*$$

so *the reference price is equal to the equilibrium price*.
For strictly positive values of the parameters three equilibria always exist:

$$P_F^* = F; \quad P_+^* = F + \sqrt{\frac{c}{f} - 1}; \quad P_-^* = F - \sqrt{\frac{c}{f} - 1}$$

Stability

By computing the eigenvalues of the Jacobian matrix of map T associated to the fundamental equilibrium P_F^* we obtain:

$$\xi_1^F = \lambda \qquad \xi_2^F = 1 + \alpha(c - f)$$

so *it is locally stable if $c < f < c + \frac{2}{\alpha}$*, otherwise it is a *saddle*.

Stability

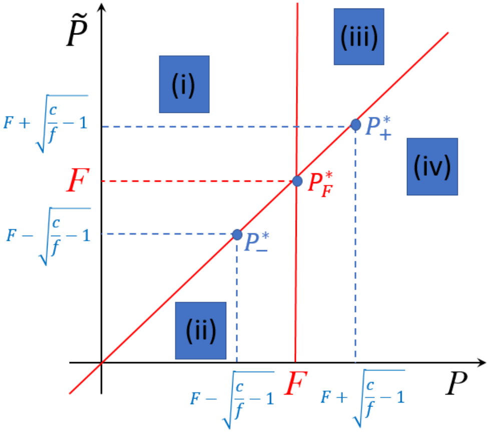
By computing the eigenvalues of the Jacobian matrix of map T associated to the fundamental equilibrium P_F^* we obtain:

$$\xi_1^F = \lambda \qquad \xi_2^F = 1 + \alpha(c - f)$$

so *it is locally stable if $c < f < c + \frac{2}{\alpha}$*, otherwise it is a *saddle*.

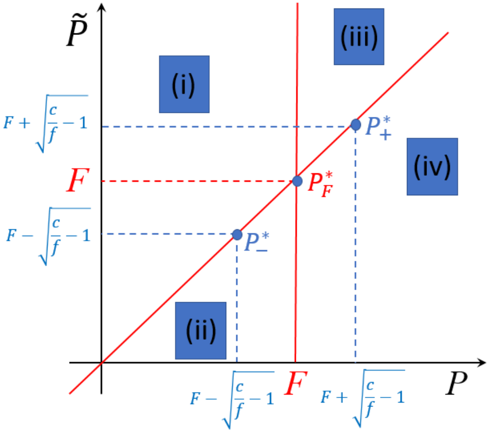
Stability

For the other two equilibria we must consider that they are located in a boundary region between two different maps:



Stability

For the other two equilibria we must consider that they are located in a boundary region between two different maps:



Loss of stability

By calculating the Jacobian matrices at the equilibria and by using the Jury conditions we find that the equilibria are locally stable if:

P^*_- $\Downarrow$

$$1-\alpha(c+f)-\frac{\lambda\alpha\sqrt{\frac{c}{f}-1}s_l}{1+\lambda}>0 \text{ (I)}$$

$$1-\alpha(c+f)+\frac{\lambda\alpha\sqrt{\frac{c}{f}-1}s_g}{1+\lambda}>0 \text{ (II)}$$

P^*_+ $\Downarrow$

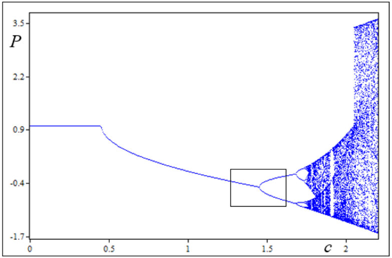
$$1-\alpha(c+f)-\frac{\lambda\alpha\sqrt{\frac{c}{f}-1}b_l}{1+\lambda}>0$$

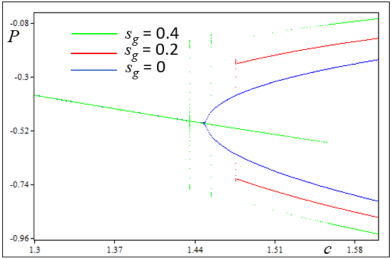
$$1-\alpha(c+f)+\frac{\lambda\alpha\sqrt{\frac{c}{f}-1}b_g}{1+\lambda}>0$$

and a Flip bifurcation may occur.
The effect of DE is ambiguous.

Loss of stability

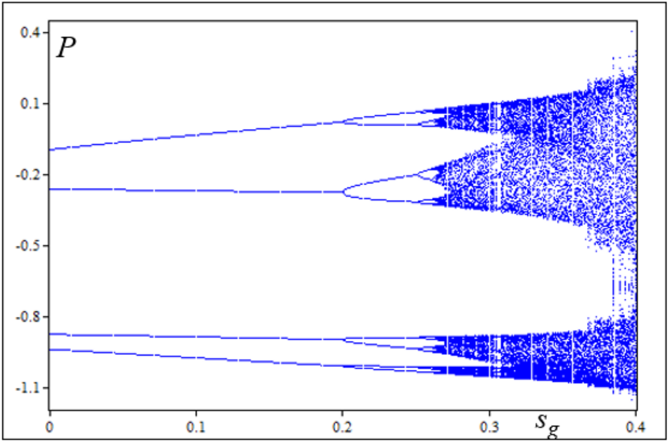
$$\alpha = 1,\ f = 0.45,\ \lambda = 0.9,\ F = 1,\ s_l = b_g = s_g/2,\ b_l = s_g$$





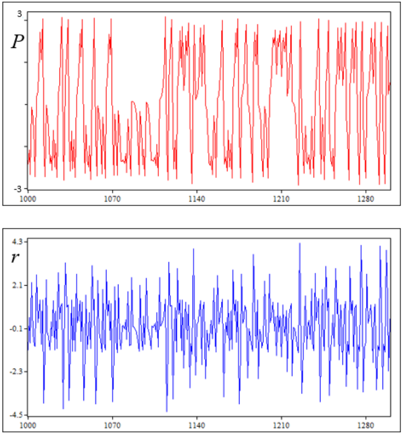
Loss of stability (chaos)

$$\alpha = 1,\ f = 0.45,\ \lambda = 0.9,\ F = 1,\ c = 1.7,\ s_l = b_g = s_g/2,\ b_l = s_g$$



Chaotic dynamics (timeseries)

$\alpha = 1,\ f = 0.45,\ \lambda = 0.9,\ F = 1,\ c = 2.18,\ s_g = 0.2,\ s_l = b_g = s_g/2,\ b_l = s$



Summary (deterministic skeleton)

- DE accelerates the occurrence of chaotic dynamics;
- Chaotic motion permits to reproduce some *qualitative stylized facts* of financial markets (bubbles and crashes, excess volatility, ecc....)

What about more quantitative features of financial markets?

A stochastic version

We introduced two stochastic elements to our model:

1. A geometric Brownian motion to the Fundamental value
2. Parameter "c" moves following a random walk

The new model

The stochastic model

$$T = \begin{cases} P' = P + \alpha \left[f(F-P) + f(F-P)^3 + c(P,\widetilde{P})(P-F) \right] \\ \widetilde{P}' = \lambda \widetilde{P} + (1-\lambda)P \\ F' = F + \xi_F \\ c' = c + \xi_c \end{cases}$$

where $\xi_F \sim N(0, \sigma_F^2)$ and $\xi_c \sim N(0, \sigma_c^2)$.

Calibration of DE

We decided to keep the parameters not related with DE fixed:

$$\alpha = 1,\ f = 0.45,\ \lambda = 0.9,\ s_l = b_g = s_g/2,\ b_l = s_g$$

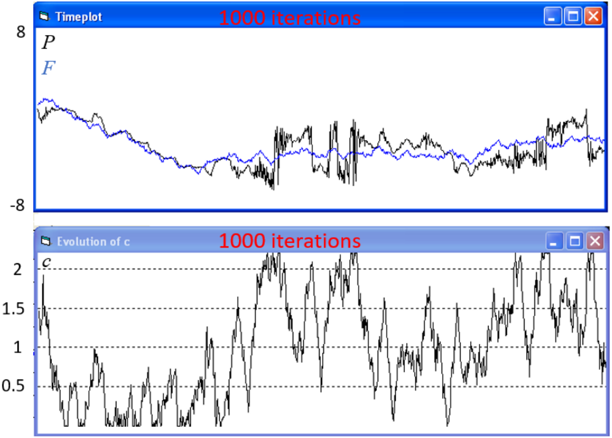
and we used $\xi_F \sim N(0, 0.015)$ and $\xi_c \sim N(0, 0.01)$.
Then we performed a Montecarlo analysis (1000 runs, each run 5000 iterations) to investigate to role of DE:

Parameter	$r_{\min}$	$r_{\max}$	V	K	ac_r^1	ac_r^5	ac_r^{10}	ac_r^{20}
$s_g = 0$	-0.031	+0.03	0.387	2.97	-0.35	-0.016	-0.035	-0.003
$s_g = 0.2$	-0.048	+0.038	0.46	5.85	-0.37	-0.015	-0.017	-0.002
$s_g = 0.3$	-0.052	+0.045	0.55	6.79	-0.35	0.01	-0.012	0.001

Parameter	$ac_{\|r\|}^1$	$ac_{\|r\|}^5$	$ac_{\|r\|}^{10}$	$ac_{\|r\|}^{20}$
$s_g = 0$	0.49	0.34	0.19	0.14
$s_g = 0.2$	0.49	0.35	0.2	0.15
$s_g = 0.3$	0.5	0.37	0.21	0.16

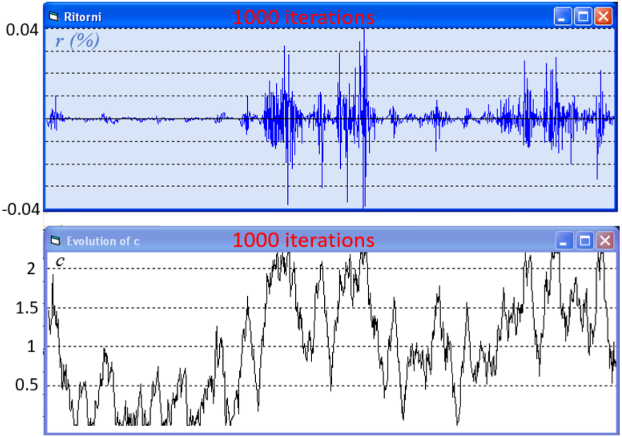
An example

$$\alpha = 1,\ f = 0.45,\ \lambda = 0.9,\ s_g = 0.2,\ s_l = b_g = s_g/2,\ b_l = s_g$$



An example

$$\alpha = 1,\ f = 0.45,\ \lambda = 0.9,\ s_g = 0.2,\ s_l = b_g = s_g/2,\ b_l = s_g$$



Summary and Conclusions

- The Disposition Effect may have deeper effects on asset prices and returns (more than just a momentum effect);
- The DE may increase the volatility of prices and returns;
- We plan to improve the analysis of the model, especially for the stochastic version.