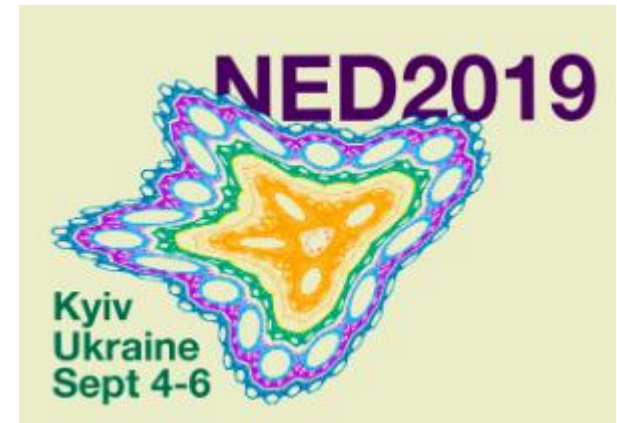
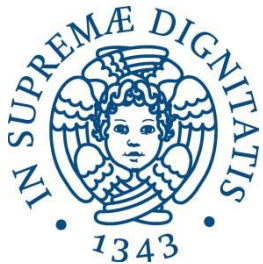


Pollution, fertility and public policies

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Kyiv, Ukraine, NED 2019, September 09th, 2016

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Motivations

- Micro and Macro founded model to study the relationship between (some) economic and environmental variables;
- Understand different development regimes;
- Study possible determinants of poverty traps of take-off.

Possible linkages between economic and environmental variables

(Caravaggio and Sodini 2018, *Frontiers in Applied Mathematics and Statistics*)

- Environment may be a (private or public) productive input in the production function (Antoci et al., *JET* 2011 and Caravaggio and Sodini, *CNSNS*, 2018);
- Environment may be a good entering the utility function (John and Pecchenino (1994), Antoci and Sodini, *CSF* 2009, Antoci et al. *FJAM* 2010, Antoci et al. *Nonlinear Dynamics in Economics, Finance and the Social Sciences*, 2010, Antoci et al. *CNSNS*, 2016);
- Climate change and Maladaptation (Antoci et al., *EDE*, 2019);
- Environment may affect the life-expectancy (Raffin and Seegmuller 2014).

- Intertemporal externalities (OLG model);
- Externalities in the optimization problem;

Changes in life-expectancy may produce changes in

- Saving;
- Fertility choices;
- Consumption;
- Investment in human capital.

This issue has been recently explored for HIV epidemics (Gori et al., MD 2019, Gori et al., forthcoming, Gori et al. WP)

Unified growth theory

- Basic elements: life-expectancy, fertility, physical and human capital accumulation are studied in a unique model with simple specifications that make the analysis possible.
- De la Croix and Doepke (2003) study a model where transition between phases is driven by heterogeneity between agents.
- Yakita (2010) introduces a simplified framework where different unexplored issues may be studied.

The model

Utility function

$$U(n_t, C_{t+1}, h_{t+1}) := \ln n_t + \beta_t [\rho \ln(h_{t+1}) + \ln(C_{t+1})]$$

$$\beta_t \in (0, 1), \rho > 0$$

n_t is the number of children;

C_{t+1} is the private (old-age) consumption;

h_{t+1} is the human capital of the offspring;

The model

Constraints

$$s_t = (1 - \tau)w_t(1 - n_t z)h_t - e_t n_t$$

$$C_{t+1} = R_{t+1} s_t$$

$$h_{t+1} := \varepsilon(h_t \theta + e_t)^{\delta} \bar{h}_t^{1-\delta} \quad e_t \geq 0$$

with $\delta \in (0, 1)$, $\tau \in (0, 1)$, $\theta > 0$, $z > 0$, $\varepsilon > 0$

Agent's problem

$$\max_{n_t > 0, e_t \geq 0} \ln n_t + \beta \left[\rho \ln \left(\varepsilon (\theta h_t + e_t)^\delta \bar{h}_t^{1-\delta} \right) + \ln \{ R_{t+1} [(1 - \tau) w_t (1 - n_t z) h_t - e_t n_t] \} \right]$$

KKT conditions

$$\beta \delta \rho < 1$$

$$\begin{aligned} \mathcal{L}(n_t, e_t, \lambda_t) \quad : \quad &= \ln n_t + \beta \left[\rho \ln \left(\varepsilon (\theta h_t + e_t)^\delta \bar{h}_t^{1-\delta} \right) + \right. \\ &\quad \left. + \ln \{ R_{t+1} [(1 - \tau) w_t (1 - n_t z) h_t - e_t n_t] \} \right] + \lambda_t e_t \end{aligned}$$

Agent's problem (I)

Solution of the optimization problem

$$\left\{ \begin{array}{l} n_t = \frac{1}{z(\beta_t + 1)} \\ e_t = 0 \\ C_{t+1} = \frac{\beta_t h_t w_t (1 - \tau)}{1 + \beta_t} R_{t+1} \end{array} \right. \quad w_t < \frac{\theta}{\rho \beta_t \delta (1 - \tau) z}$$

$$\left\{ \begin{array}{l} n_t = \frac{w_t (1 - \tau) (1 - \beta_t \delta \rho)}{(1 + \beta_t) (z (1 - \tau) w_t - \theta)} \\ e_t = \frac{(z w_t \beta_t \rho (1 - \tau) \delta - \theta)}{(1 - \beta_t \delta \rho)} h_t \\ C_{t+1} = \frac{\beta_t h_t w_t (1 - \tau)}{1 + \beta_t} R_{t+1} \end{array} \right. \quad w_t \geq \frac{\theta}{\rho \beta_t \delta (1 - \tau) z}$$

Agent's problem (II)

Solution of the optimization problem

$$s_t = \frac{w_t h_t \beta_t (1 - \tau)}{\beta_t + 1}$$

Life-expectancy

$$\beta_t = \frac{b_0 + \frac{G_t^1}{P_t}}{b_1 + \frac{G_t^1}{P_t}}$$

where G_t^1 is the public expenditure in health, P_t is the pollution level at t
 $\frac{b_0}{b_1} (< 1)$ is the life expectancy if the ratio $\frac{G_t^1}{P_t} = 0$.

Life-expectancy

β_t is endogenous and its level is determined by the following equation

$$\beta_t := \frac{(b_0 + G_t^1)}{b_1 + G_t^1 + P_t} = \frac{1 + \frac{b_0}{G_t^1}}{1 + \frac{b_1}{G_t^1} + \frac{P_t}{G_t^1}}$$

where G_t^1 is the public expenditure in health, P_t is the pollution level at t
 $\frac{b_0}{b_1} (< 1)$ is the life expectancy without pollution and public expenditure.

Pollution Dynamics

$$P_{t+1} = (1 - \sigma)P_t + \gamma Y_t - \phi G_t^2$$

where $\sigma \in [0, 1]$, $\gamma > 0$, $\phi > 0$.

Public expenditure

$$G_t^1 = \tau_1 L_t w_t$$

$$G_t^2 = \tau_2 L_t w_t$$

where $L_t := N_t(1 - n_t z)h_t$ is the effective labor and $\tau_1 + \tau_2 = \tau$

Production and prices

$$Y_t = AK_t^\alpha L_t^{1-\alpha}$$

$$w_t = \frac{A(1-\alpha)K_t^\alpha}{(N_t(1-n_tz)h_t)^\alpha}$$

$$R_t = \frac{A\alpha K_t^{\alpha-1}}{(N_t(1-n_tz)h_t)^{\alpha-1}}$$

Population Dynamics

$$N_{t+1} = n_t N_t$$

if $n_t < 1 \forall t > \bar{t}$ then $N_t \rightarrow 0$;

if $n_t > 1 \forall t > \bar{t}$ then $N_t \rightarrow +\infty$

Market Equilibrium Conditions

$$K_{t+1} = s_t N_t$$

Equilibrium Dynamics

Because of the returns in human capital accumulation, it is possible that $K_t \rightarrow +\infty$ and $h_t \rightarrow +\infty$. We introduce the following change of variable.

$$K_{t+1} = s_t N_t$$

According to the different regimes we have:

$$n_t = \frac{w_t(1 - \tau)(1 - \beta_t \delta \rho)}{(1 + \beta_t)(z(1 - \tau)w_t - \theta)}$$
$$w_t = \frac{A(1 - \alpha)K_t^\alpha}{(N_t(1 - n_t z)h_t)^\alpha}$$

or

$$n_t = \frac{1}{(1 + \beta_t)(z(1 - \tau)w_t - \theta)}$$
$$w_t = \frac{A(1 - \alpha)K_t^\alpha}{(N_t(1 - n_t z)h_t)^\alpha}$$

Equilibrium Dynamics (First Stage)

$$k_{t+1} = \frac{w_t(1 - \tau)(1 - n_t z)}{n_t \varepsilon \theta^\delta}$$

$$p_{t+1} = \frac{A k_t^\alpha (1 - \alpha)(\gamma - \phi \tau_2)(1 - n_t z)^{1 - \alpha} + p_t(1 - \sigma)}{n_t \varepsilon \theta^\delta}$$

Equilibrium Dynamics (Second Stage)

$$k_{t+1} = \frac{\theta n_t + Ak_t^\alpha (1 - \tau)(1 - \beta_t \delta \rho - n_t z)(1 - \alpha)(1 - n_t z)^{-\alpha}}{(1 - \beta_t \delta \rho) n_t \varepsilon \left(\frac{\delta \rho \beta_t (z(1 - \tau) w_t - \theta)}{1 - \beta_t \delta \rho} \right)^\delta}$$
$$p_{t+1} = \frac{Ak_t^\alpha (1 - \alpha)(\gamma_1 - \phi \tau_2)(1 - n_t z)^{1 - \alpha} - p_t(1 - \sigma)}{n_t \varepsilon \left[\frac{z w_t \rho \beta_t (1 - \tau) \delta - \theta}{1 - \beta_t \delta \rho} \right]^\delta}$$

Preliminary results

- Multiple attracting fixed points;
- The mix of policies may be determinant in defining the trajectories of the economy
- Changes of policies driven by environmental target may change the long run dynamics
- Cycles?

Thank you