Pollution, fertility and public policies

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Motivations

- Micro and Macro founded model to study the relationship between (some) economic and environmental variables;
- Understand different development regimes;
- Study possible determinants of poverty traps of take-off.

Possible linkages between economic and environmental variables (Caravaggio and Sodini 2018, Frontiers in Applied Mathematics and Statistics)

- Environment may be a (private or public) productive input in the production function (Antoci et al., JET 2011 and Caravaggio and Sodini, CNSNS, 2018);
- Environment may be a good entering the utility function (John and Pecchenino (1994), Antoci and Sodini, CSF 2009, Antoci et al. FJAM 2010, Antoci et al. Nonlinear Dynamics in Economics, Finance and the Social Sciences, 2010, Antoci et al. CNSNS, 2016);
- Climate change and Maladaptation (Antoci et al., EDE, 2019);
- Environment may affect the life-expectancy (Raffin and Seegmuller 2014).

- Intertemporal externalities (OLG model);
- Externalities in the optimization problem;

Changes in life-expectancy may produce changes in

- Saving;
- Fertility choices;
- Consumption;
- Investment in human capital.

This issue has been recently explored for HIV epidemics (Gori et al., MD 2019, Gori et al., forthcoming, Gori et al. WP)

Unified growth theory

- Basic elements: life-expectancy, fertility, physical and human capital accumulation are studied in a unique model with simple specifications that make the analysis possible.
- De la Croix and Doepke (2003) study a model where transition between phases is driven by heterogeneity between agents.
- Yakita (2010) introduces a simplified framework where different unexplored issues may be studied.

The model

Utility function

$$U(n_t, C_{t+1}, h_{t+1}) := \ln n_t + \beta_t \left[\rho \ln(h_{t+1}) + \ln(C_{t+1})\right]$$

$$\beta_t \in (0,1),\ \rho > 0$$

n_t is the number of children;
C_{t+1} is the private (old-age) consumption;
h_{t+1} is the human capital of the offspring;

The model

Constraints

$$s_t = (1-\tau)w_t(1-n_t z)h_t - e_t n_t$$

$$C_{t+1} = R_{t+1} s_t$$

$$h_{t+1} := \varepsilon(h_t\theta + e_t)^\delta \overline{h}_t^{1-\delta} \qquad e_t \geq 0$$

with $\delta \in (0,1)$, $\tau \in (0,1)$, $\theta > 0$, $z > 0$, $\varepsilon > 0$

Agent's problem

$$\max_{n_t > 0, e_t \geq 0} \ln n_t + \beta \left[\rho \ln \left(\varepsilon (\theta h_t + et)^{\delta} \overline{h}_t^{1-\delta} \right) + \ln \left\{ R_{t+1} \left[(1-\tau) w_t (1 - n_t z) h_t - e_t n_t \right] \right\} \right]$$

KKT conditions

$$\beta \delta \rho < 1$$

$$\begin{aligned} \mathcal{L}(n_t, e_t, \lambda_t) \quad : \quad &= \ln n_t + \beta \left[\rho \ln \left(\varepsilon (\theta h_t + et)^{\delta} \overline{h}_t^{1-\delta} \right) + \right. \\ &\left. + \ln \left\{ R_{t+1} \left[(1-\tau) w_t (1 - n_t z) h_t - e_t n_t \right] \right\} \right] + \lambda_t e_t \end{aligned}$$

Agent's problem (I)

Solution of the optimization problem

$$\begin{cases} n_t = \frac{1}{z(\beta_t+1)} \\ e_t = 0 \\ C_{t+1} = \frac{\beta_t h_t w_t (1-\tau)}{1+\beta_t} R_{t+1} \end{cases} \quad w_t < \frac{\theta}{\rho\beta_t\delta(1-\tau)z}$$

$$\begin{cases} n_t = \frac{w_t(1-\tau)(1-\beta_t\delta\rho)}{(1+\beta_t)(z(1-\tau)w_t-\theta))} \\ e_t = \frac{(zw_t\beta_t\rho(1-\tau)\delta-\theta)}{(1-\beta_t\delta\rho)} h_t \\ C_{t+1} = \frac{\beta_t h_t w_t (1-\tau)}{1+\beta_t} R_{t+1} \end{cases} \quad w_t \geq \frac{\theta}{\rho\beta_t\delta(1-\tau)z}$$

Agent's problem (II)

Solution of the optimization problem

$$s_t = \frac{w_t h_t \beta_t (1 - \tau)}{\beta_t + 1}$$

Life-expectancy

$$\beta_t = \frac{b_0 + \frac{G_t^1}{P_t}}{b_1 + \frac{G_t^1}{P_t}}$$

where G_t^1 is the public expenditure in health, P_t is the pollution level at t $\frac{b_0}{b_1}(< 1)$ is the life expectancy if the ratio $\frac{G_t^1}{P_t} = 0$.

Life-expectancy

β_t is endogenous and its level is determined by the following equation

$$\beta_t := \frac{(b_0 + G_t^1)}{b_1 + G_t^1 + P_t} = \frac{1 + \frac{b_0}{G_t^1}}{1 + \frac{b_1}{G_t^1} + \frac{P_t}{G_t^1}}$$

where G_t^1 is the public expenditure in health, P_t is the pollution level at t $\frac{b_0}{b_1}(< 1)$ is the life expectancy without pollution and public expenditure.

Pollution Dynamics

$$P_{t+1} = (1 - \sigma)P_t + \gamma Y_t - \phi G_t^2$$

where $\sigma \in [0, 1]$, $\gamma > 0$, $\phi > 0$.

Public expenditure

$$\begin{aligned} G_t^1 &= \tau_1 L_t w_t \\ G_t^2 &= \tau_2 L_t w_t \end{aligned}$$

where $L_t := N_t(1 - n_t z)h_t$ is the effective labor and $\tau_1 + \tau_2 = \tau$

Production and prices

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$$

$$\begin{aligned} w_t &= \frac{A(1-\alpha)K_t^{\alpha}}{(N_t(1-n_t z)h_t)^{\alpha}} \\ R_t &= \frac{A\alpha K_t^{\alpha-1}}{(N_t(1-n_t z)h_t)^{\alpha-1}} \end{aligned}$$

Population Dynamics

$$N_{t+1} = n_t N_t$$

$$\text{if } n_t < 1 \; \forall t > \bar{t} \text{ then } N_t \to 0;$$

$$\text{if } n_t > 1 \; \forall t > \bar{t} \text{ then } N_t \to +\infty$$

Market Equilibrium Conditions

$$K_{t+1} = s_t N_t$$

Equilibrium Dynamics

Because of the returns in human capital accumulation, it is possible that $K_t \to +\infty$ and $h_t \to +\infty$. We introduce the following change of variable.

$$K_{t+1} = s_t N_t$$

According to the different regimes we have:

$$
\begin{aligned}
n_t &= \frac{w_t(1-\tau)(1-\beta_t\delta\rho)}{(1+\beta_t)(z(1-\tau)w_t-\theta))} \\
w_t &= \frac{A(1-\alpha)K_t^{\alpha}}{(N_t(1-n_t z)h_t)^{\alpha}}
\end{aligned}
$$

or

$$
\begin{aligned}
n_t &= \frac{1}{(1+\beta_t)(z(1-\tau)w_t-\theta))} \\
w_t &= \frac{A(1-\alpha)K_t^{\alpha}}{(N_t(1-n_t z)h_t)^{\alpha}}
\end{aligned}
$$

Equilibrium Dynamics (First Stage)

$$k_{t+1} = \frac{w_t(1-\tau)(1-n_t z)}{n_t \varepsilon \theta^{\delta}}$$

$$p_{t+1} = \frac{A k_t^{\alpha}(1-\alpha)(\gamma - \phi\tau_2)(1-n_t z)^{1-\alpha} + p_t(1-\sigma)}{n_t \varepsilon \theta^{\delta}}$$

Equilibrium Dynamics (Second Stage)

$$k_{t+1} = \frac{\theta n_t + A k_t^{\alpha}(1-\tau)(1-\beta_t\delta\rho - n_t z)(1-\alpha)(1-n_t z)^{-\alpha}}{(1-\beta_t\delta\rho)n_t\varepsilon\left(\frac{\delta\rho\beta_t(z(1-\tau)w_t-\theta)}{1-\beta_t\delta\rho}\right)^{\delta}}$$

$$p_{t+1} = \frac{A k_t^{\alpha}(1-\alpha)(\gamma_1-\phi\tau_2)(1-n_t z)^{1-\alpha} - p_t(1-\sigma)}{n_t\varepsilon\left[\frac{z w_t\rho\beta_t(1-\tau)\delta-\theta}{1-\beta_t\delta\rho}\right]^{\delta}}$$

Preliminar results

- Multiple attracting fixed points;
- The mix of policies may be determinant in defining the trajectories of the economy
- Changes of policies driven by environmental target may change the long run dynamics
- Cycles?

Thank you