Employment fluctuations in noisy signaling labor market

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Contents

What is "signaling dynamics"?

- Signaling dynamics describes interactions between senders and receivers over time.
- Noldeke and Van Damme (1990) analyzes a multiperiod version of the Spence's job market signalling mode.
- Noldeke and Samuelson (1997) introduce pertabations into Spence's dynamic model and examines the condition to chose one between multiple equilibria.

Signaling dynamics in a labor market

- The jobseekers send signals to reveal their true type and the employers decide to employ whom and how many for given observations of the job seekers' signaling.
- The signaling is noisy so that the employers cannot figure out the job seekers true type from the signaling.
- Heinsalu (2018) studies costly signaling model in which the signaling effort is chosen in multiple periods and observed with noise.
- The employers decide their employment level by solving their profit maximization problem.

Summary of previous research

- Analyzes the existence of two types of pooling equilibrium and unique separating equilibrium when there are two types of job seekers with different productivity and signaling costs.
- Proves analytically that the above equilibria coexist in any combination.
- Examine numerically the multistability of those equilibria and periodic fluctuations of the employment level.

Aim of this paper

- Extend the model to cases where signaling costs are continuously distributed.
- Analyze existence conditions of signaling equilibria.
- Numerically examine the local stability of signaling equilibria and demonstrate complex fluctuations of employment level with the distribution of signaling costs.

Optimal employment choice of the firm

Production output level y depends on employment level x and productivity of employed workers. Then,

$$y \equiv \sqrt{\frac{1+\alpha}{2}x}$$

Here α is the lowest productivity of employed workers. For given market price of the production p and wage w for the workers, the firm choses employment level x to maximize the profit pi given by

$$\pi(x) = p \cdot y - \frac{1+\alpha}{2}w$$

Here $d\pi/dx \lesseqgtr 0$ and $d^2\pi/dx^2 < 0$. Solving the optimization problem of the firm and normarizing p to 1, we derive a reaction function f given by

$$f(\alpha) \equiv \arg\max \pi(x) = \frac{1}{2w^2(1+\alpha)}$$

Expectation of job seekers' productivity

From the assumption of noisy signaling, the firm cannot recognize exactly the true productivity of job seekers by observing the signals. Let e denote the signaling level of job seekers which means a proportion of job seekers who send a signal.($0 \leq e \leq 1$) So that firm takes an expectation on α for given observation of signaling level e. We assume the relation between α and e that α is increasing function of the signaling level e.
Firm's expectation on job seekers' productivity:

$$g(e) \equiv a_0 + (a_1 - a_0)e, \quad 0 < a_0 < a_1 \leq 1.$$

Employment policy of the firm

Let M denote the population of job seekers. Firm's employment choice as a reaction toward job seekers' signaling includes the profit maximization of the firm. Then, the employment level x may not be equal to the amount of signaling job seekers eM.
The firm shows following employment policy to job seekers.

- The firm employs signaling job seekers preferentially.
- If $x < eM$, only a part of signaling job seekers are employed and no non-signaling job seekers are not employed.
- If $x > eM$, all of signaling job seekers are employed and some of non-signaling job seekers are employed within the excess amount.

Effectiveness of signaling

Probability that signaling job seekers are employed:

$$\rho_s = \begin{cases} \dfrac{x}{eM} & \text{if} \quad x < eM, \\ 1 & \text{if} \quad x \geq eM. \end{cases}$$

Probability that non-signaling job seekers are employed:

$$\rho_n = \begin{cases} 0 & \text{if} \quad x < eM, \\ \dfrac{x - eM}{(1-e)M} & \text{if} \quad x \geq eM. \end{cases}$$

Distribution of signaling cost

Suppose that signaling costs of job seekers are uniformly distributed in the finite interval $[c_L, c_H]$ $(0 \leq c_L < c_H)$. Then the cumulative distribution function of signaling cost, $CDF(c)$ is given by

$$CDF(c) \equiv \begin{cases} 0 & \text{if} \quad c < c_L \\ \dfrac{c - c_L}{c_H - c_L} & \text{if} \quad c_L \leq c \leq c_H \\ 1 & \text{if} \quad c > c_H. \end{cases}$$

Observation of signaling level

Job seekers send a signal if the expected benefit of signaling σ exceeds their signaling cost where

$$\sigma = p_s w - p_n \alpha w.$$

Then we denote h as the entire signaling level,

$$h(x, \alpha, e) \equiv \begin{cases} 0 & \text{if} \quad \sigma < c_L, \\ \dfrac{\sigma - c_L}{c_H - c_L} & \text{if} \quad c_L \leq \sigma \leq c_H, \\ 1 & \text{if} \quad \sigma > c_H. \end{cases}$$

3-Dimensional dynamics of signaling market

Summarizing the discussion above finally we construct the 3-dimentional dynamical system as the signaling dynamics.

$$
\begin{aligned}
x_{t+1} &= f(\alpha_t) \\
\alpha_{t+1} &= g(e_t) \\
e_{t+1} &= h(x_t, \alpha_t, e_t).
\end{aligned}
$$

Signaling equilibria

In this model following three signaling equilibria could exist.

- Pooling equilibrium without signaling: no job seeker sends a signal ($e = 0$).
- Separating equilibrium: some of job seekers send a signal ($0 < e < 1$).
- Pooling equilibrium with signaling: both of high- and low-productive job seekers send a signal ($e = 1$).

Fixed points at pooling equilibria

A fixed point of the dynamical system corresponds to a signaling equilibrium. Let s_i denote fixed points at pooling equilibria ($i \in \{0, 1\}$). Then

$$\begin{aligned} s_0 &= (x_0^*, \alpha_0^*, 0), \\ s_1 &= (x_1^*, \alpha_1^*, 1), \end{aligned}$$

where

$$x_0^* = \frac{1}{2(a_0 + 1)w^2},$$

$$x_1^* = \frac{1}{2(a_1 + 1)w^2},$$

$$\alpha_0 = a_0, \quad \text{and} \quad \alpha_1 = a_1.$$

Boundary conditions of pooling equilibria without signaling

Solving the condition $\sigma = c_L$, we get

$$x = \frac{c_L M}{w} e$$

for $x < eM$ and get

$$x = \frac{M((a_1 - a_0)w + w - c_L)e + a_0 w - w + c_L)}{(a_1 - a_0)we + a_0 w}$$

for $x > eM$.

Boundary conditions of pooling equilibria with signaling

Solving the condition $\sigma = c_H$, we get

$$x = \frac{c_H M}{w} e$$

for $x < eM$ and get

$$x = \frac{M((a_1 - a_0)w + w - c_H)e + a_0 w - w + c_H)}{(a_1 - a_0)we + a_0 w}$$

for $x > eM$.

Phase diagram

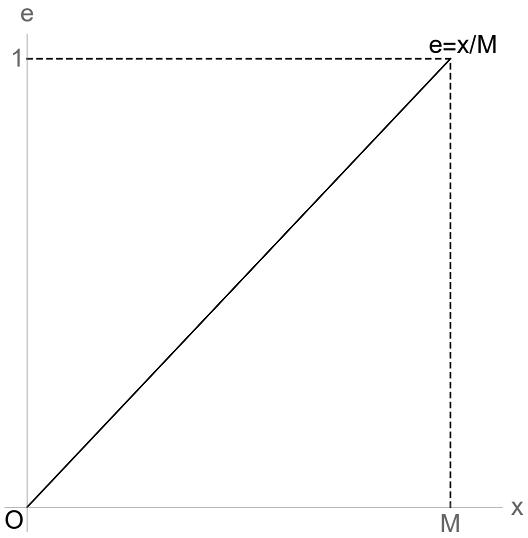
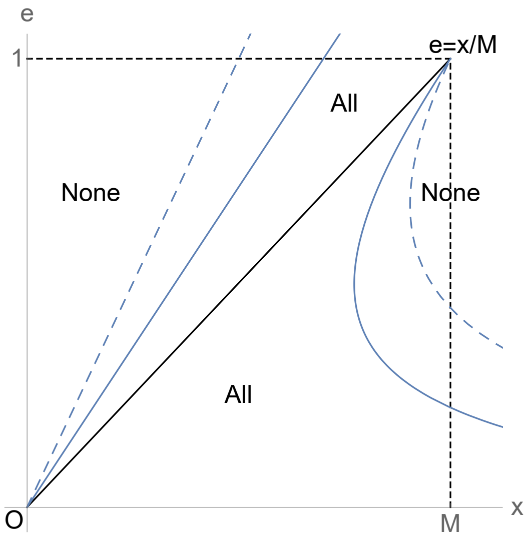


Figure: Reaction functions and an equilibrium

Phase diagram



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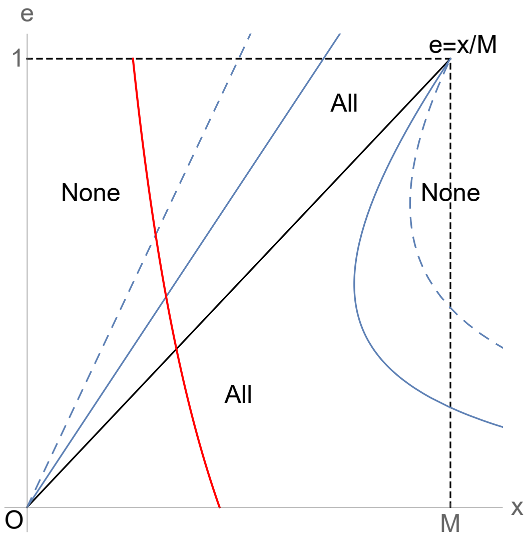

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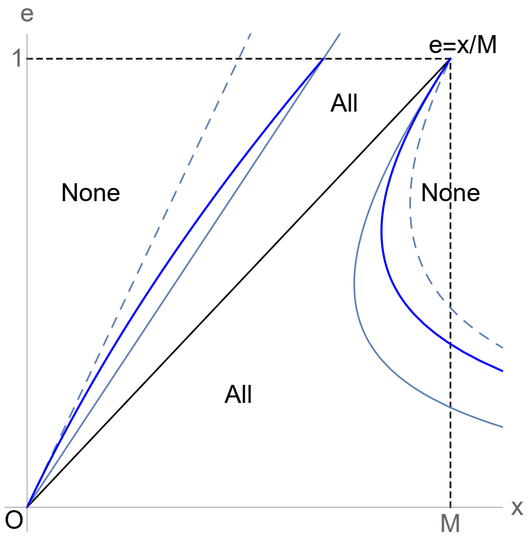

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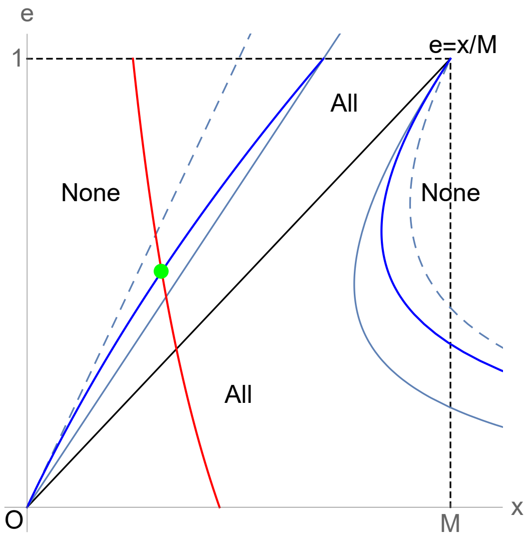


Figure: Reaction functions and an equilibrium

Pooling equilibrium with signaling

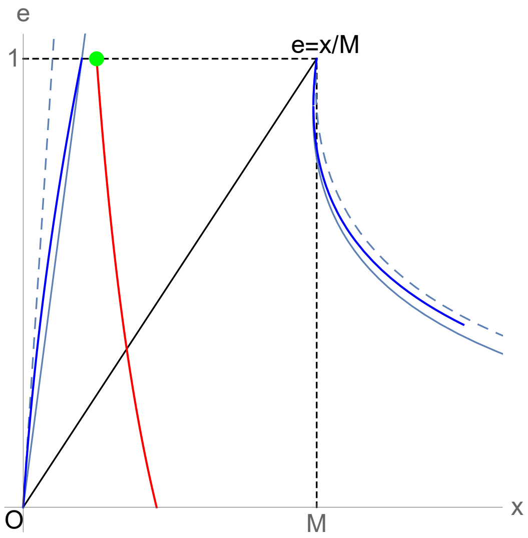


Figure: Unique pooling equilibrium with signaling ($c_L = 0.1$ and $c_H = 0.2$)

Existence of signaling equilibrium 1

Proposition 3.1

For $x < eM$, a pooling equilibrium with signaling s_1 exists if $c_H < \hat{c}_H$ and a separating equilibrium exists if $c_H \geq \hat{c}_H$ where

$$\hat{c}_H = \frac{2(1 - a_0^2)Mw^2 + a_0}{2(a_0 + 1))Mw}.$$

Pooling equilibrium with signaling

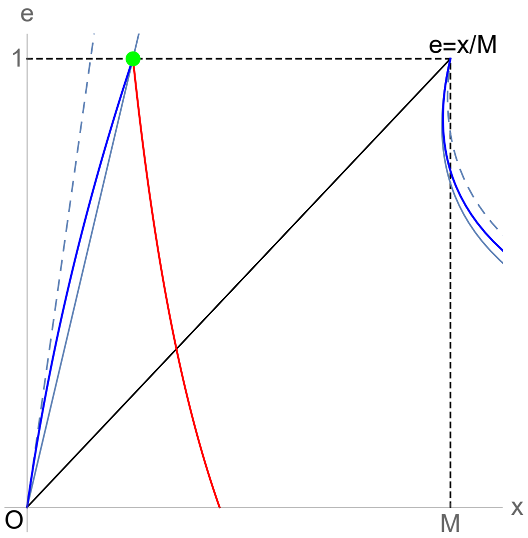


Figure: Unique pooling equilibrium with signaling ($c_L < \hat{c}_H$ and $c_H = \hat{c}_H$)

Multiple separating equilibria 1

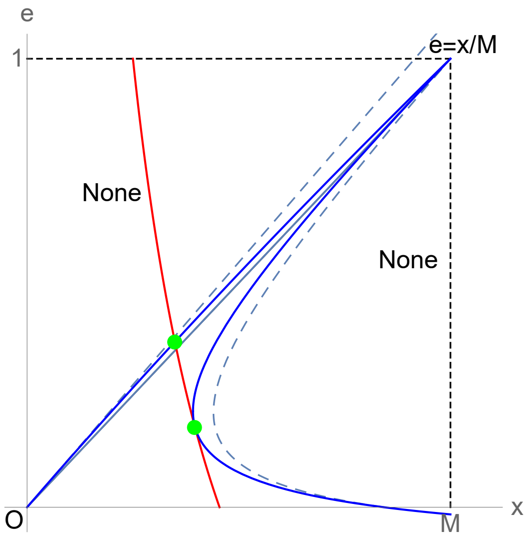


Figure: Separating equilibria ($c_L \simeq 0.66619$ and $c_H = 1$)

Multiple separating equilibria 2

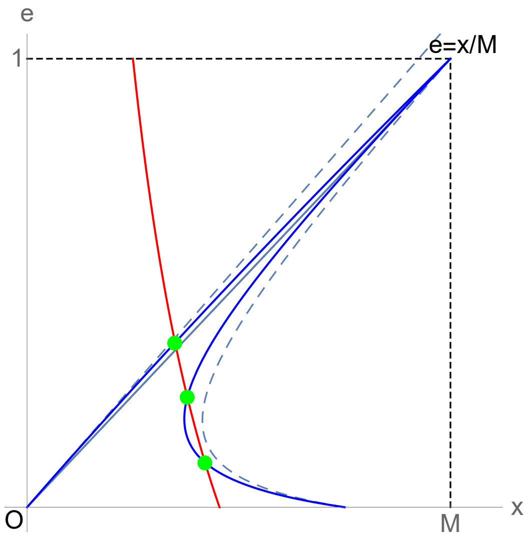


Figure: Coexitence of 3 separating equilibria ($c_L = 0.925$ and $c_H = 1$)

Existence of signaling equilibrium 2

Proposition 3.2

For $x > eM$, a pooling equilibrium without signaling s_0 exists if $c_L > \hat{c}_L$ where

$$\hat{c}_L = \frac{1}{2a_1 + 2)wM)}.$$

Pooling equilibrium without signaling

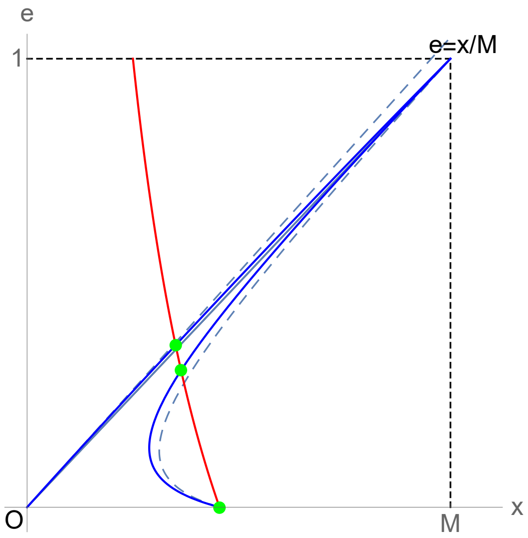


Figure: Coexistence of separating and non-signaling pooling equilibrium ($c_L < \hat{c}_L$ and $c_H > \hat{c}_L$)

Existence conditions of signaling equilibria

Assuming that $c_L < c_H$,

- If $c_H \leq \hat{c}_H$, pooling equilibrium s_1 exists in $\{(x, e) | x < eM\}$ and is unique.
- If $c_H > \hat{c}_H$, separating equilibrium exists in $\{(x, e) | x < eM\}$ and is unique.
- In $\{(x, e) | x > eM\}$, separating equilibria co-exist.
- If $c_L \geq \hat{c}_L$, pooling equilibrium s_0 and separating equilibrium co-exist in $\{(x, e) | x > eM\}$.

Existence conditions of signaling equilibria

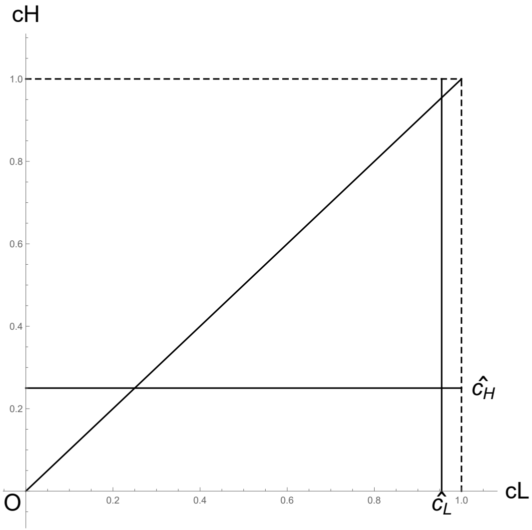


Figure: Existence conditions of signaling equilibria with $c_L$and c_H

Existence conditions of signaling equilibria

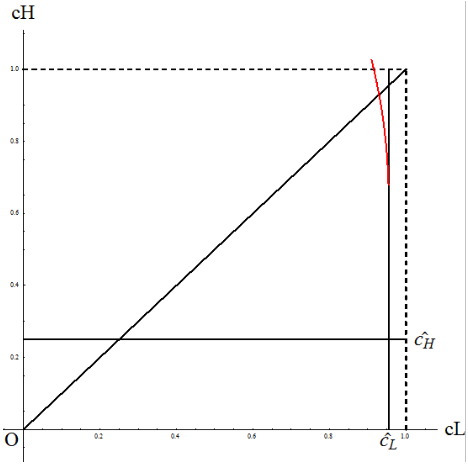


Figure: Existence conditions of signaling equilibria with $c_L$and c_H

Local stabilities of stationary points

Let J^* denotes a Jacobian matrix of signaling dynamics evaluated at a stationary point s_i,

$$J_i^* \equiv \begin{pmatrix} 0 & f'(\alpha_i^*) & 0 \\ 0 & 0 & g'(e_i^*) \\ \frac{\partial h}{\partial x_i^*} & \frac{\partial h}{\partial \alpha_i^*} & \frac{\partial h_i^*}{\partial e_i^*} \end{pmatrix}.$$

Suppose that the eigenvalues λ_n solves following characteristic polynomial,

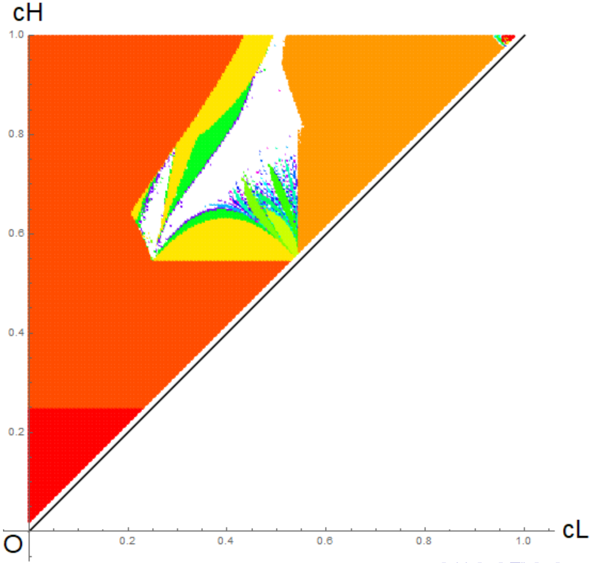
$$\begin{aligned} \det(\lambda I - J^*) &= -\lambda_n^3 - \frac{\partial h}{\partial e_i^*}\lambda_n^2 - g'(e_i^*)\frac{\partial h}{\partial \alpha_i^*}\lambda_n + f'(\alpha_i^*)g'(e_i^*)\frac{\partial h}{\partial x_i^*} \\ &= 0. \end{aligned}$$

Then, s_i is locally stable if all of λ_n are real or complex numbers with absolute value strictly less than 1.

Numerical experiments

- Examine local stability of the fixed points and bifurcations with variation of c_L and c_H.
- Other parameters are $w = 1$, $M = 1$, $a_0 = 0.1$ and $a_1 = 1$.
- Demonstrate basins of attractions in the case of multistability.

Local stabilities of stationary points with signaling costs c_L and c_H



Local stabilities of stationary points with signaling costs c_L and c_H

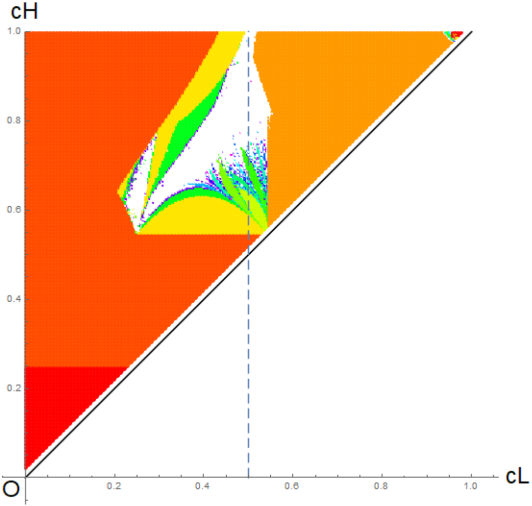


Figure: Bifurcation of signaling dynamics with c_L and c_H

Local stabilities of stationary points with signaling costs c_L and c_H

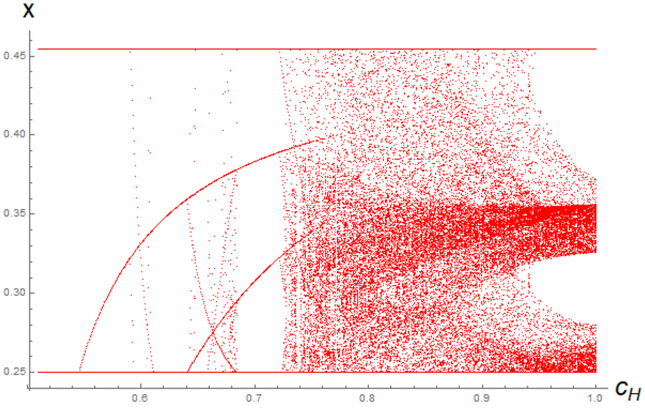


Figure: Bifurcations of employment level x with c_H ($c_L = 0.2$)

Local stabilities of stationary points with signaling costs c_L and c_H

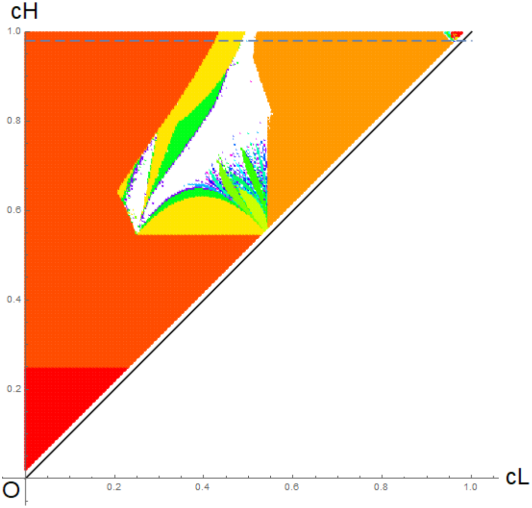


Figure: Bifurcation of signaling dynamics with c_L and c_H

Local stabilities of stationary points with signaling costs c_L and c_H

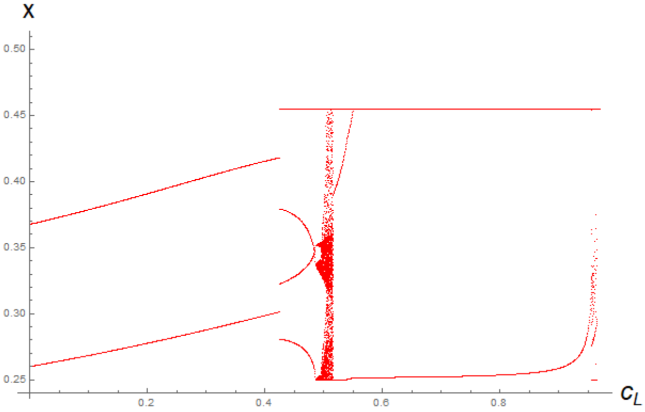


Figure: Bifurcations of employment level x with c_L ($c_H = 0.98$)

Local stabilities of stationary points with signaling costs c_L and c_H

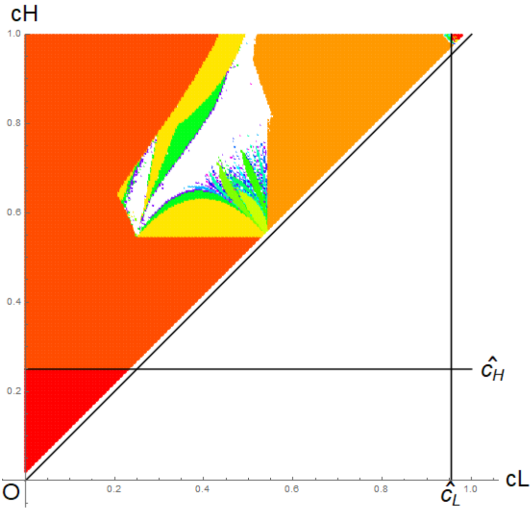


Figure: Bifurcation of signaling dynamics with c_L and c_H

Local stabilities of stationary points with signaling costs c_L and c_H

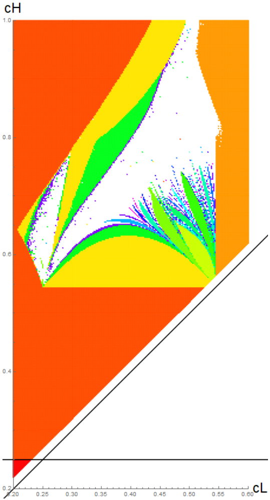


Figure: Bifurcation of signaling dynamics with c_L and c_H

Local stabilities of stationary points with signaling costs c_L and c_H

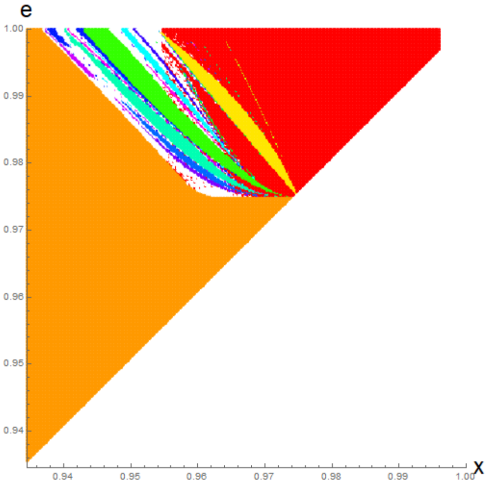


Figure: Bifurcation of signaling dynamics with c_L and c_H

Multi-stability of pooling equilibrium and periodic cycles

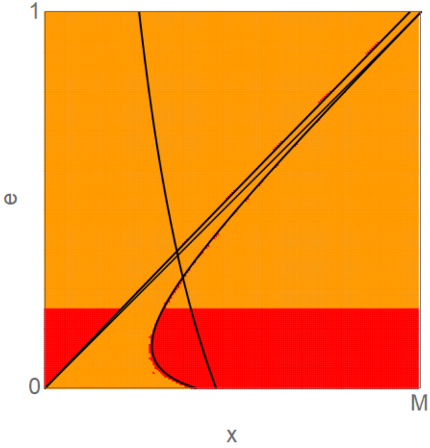


Figure: Basins of attractions with $c_L = 0.96$ and $c_H = 0.97$

Multi-stability of pooling equilibrium and periodic cycles

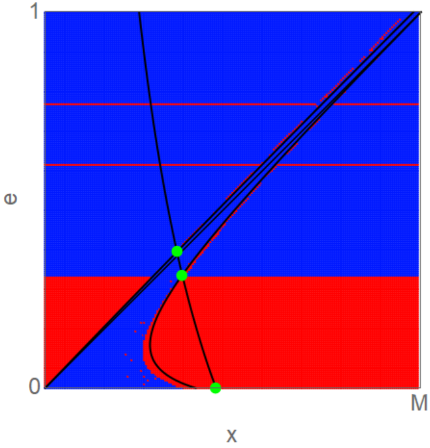


Figure: Basins of attractions with $c_L = 0.96$ and $c_H = 0.975$

Multi-stability of pooling equilibrium and periodic cycles

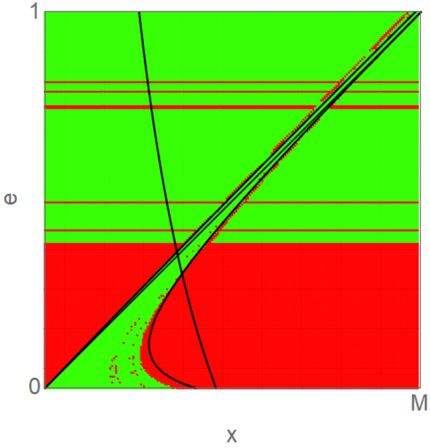


Figure: Basins of attractions with $c_L = 0.96$ and $c_H = 0.985$

Results of numerical experiments

- Pooling equilibrium with signaling s_1 ($e^* = 1$) is alway locally stable if it exists.
- For $x < eM$, unique separating equilibrium emarge if $c_H > \hat{c}_H$ and is always locally unstable.
- The signaling dynamics shows complex fluctuations of the employment level for various parameters' combination of c_L and c_H.
- If pooling equilibrium without signaling s_0 ($e^* = 0$) is locally stable, multi-stability is obserbed.

Concluding remarks

- Signaling dynamics with a continuous distribution of job seekers' signaling cost can have all of 3 types of equilibria and show complex fluctuations of employment level.
- Pooling equilibrium <u>WITH</u> signaling is unique and locally stable if it exists.
- Pooling equilibrium <u>WITHOUT</u> signaling is co-existing with separating equilibria and is locally stable while the separating equilibria are unstable.
- In a case of multiple equilibria, the signaling dynamics converges to different stationary point or a periodic cycle depending on its initial point.

Thank You!

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