The Rationality Bias

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Motivation

- Growing consensus that models with homogeneous rational expectations cannot adequately approximate actual human behavior at microeconomic or macroeconomic level
- There may be **some** highly sophisticated and well informed economic actors, but a **non-negligible fraction** of the population might **not be nearly as rational** as assumed in theoretical models
- The presence of agents with different levels of cognitive ability may lead to **non-trivial interactions**

Motivation

- Different strands of literature incorporating bounded rationality have gained popularity
- One strand incorporates **heterogeneous, possibly non-rational expectations** in macro models (Branch and McGough (2016))
- Empirical evidence suggests:
  - expectations are indeed heterogeneous
  - a sizable fraction of the population seems to follow **simple backward-looking heuristics**

  (Branch, 2004; Cornea-Madeira et al., 2017; Assenza et al., 2014; Pfajfar and Žakelj, 2016, Fuhrer, 2017)
- However, focus of theoretical work has been on aggregate dynamics, largely ignoring dynamics at the **individual level**

Contribution

- We explicitly consider how differences in cognitive ability lead to differences in consumption and wealth
- Micro-founded model with given fraction of agents **fully rational** (in conventional sense) and other fraction **boundedly rational**
- We keep track of the individual bond holdings of both groups
- **"Rationality bias"** of boundedly rational agents is driver of consumption and wealth heterogeneity
- Strong interaction between size of **rationality bias and monetary policy**

Assumptions on heterogeneous rationality

Rational households

- are fully aware of the presence of boundedly rational households
- make **model-consistent predictions** over the entire paths of output, government spending, taxes, real interest rates and preference shocks until infinity
- $E_t^R x_{t+1} = E_t x_{t+1}$

Rational consumption

$$c_t^R = \zeta \hat{b}_{t-1}^R + \zeta \bar{b}(i_{t-1} - \pi_t) + \zeta \beta E_t \sum_{s=t}^{\infty} \beta^{s-t} [\Gamma_y y_s - \Gamma_g \hat{g}_s - \Gamma_\tau \tau_s]$$
$$- \frac{(1 - \zeta \bar{b} \sigma)\beta}{\sigma} E_t \sum_{s=t}^{\infty} \beta^{s-t} (i_s - \pi_{s+1}) + \frac{\beta}{\sigma} E_t \sum_{s=t}^{\infty} \beta^{s-t} (\upsilon_s - \upsilon_{s+1}) \quad (1)$$

Assumptions on heterogeneous rationality

Boundedly rational households

- Use **Euler learning** following Branch and McGough (2009)
- **Naive expectations**, i.e. $E_t^B x_{t+1} = x_{t-1}$ (Branch, 2004 and Cornea-Madeira et al., 2017)
- Believe that all other agents form the same expectations as they do

Boundedly rational consumption

- Can iterate their Euler equation until period N, the "long-run" (can be infinite)
- Know that market clearing holds
- Believe that when they have more current wealth than the average, they will expect to able to consume more than the average in the "long-run"

$$c_t^B = \frac{1}{1-\bar{g}}(y_{t-1} - \hat{g}_{t-1}) + \psi(\hat{b}_{t-1}^B - \hat{b}_{t-1}) - \frac{1}{\sigma}[i_t - \pi_{t-1} - \upsilon_t + \upsilon_{t-1}] \quad (2)$$

Aggregate Demand

Using market clearing, **aggregate demand** can be derived as

$$
\begin{aligned}
y_t = {} & \Phi_1 E_t y_{t+1} + \Phi_2 y_{t-1} + \Phi_3 E_t \pi_{t+1} - \Phi_4 \pi_t + \Phi_5 \pi_{t-1} + \Phi_6 i_{t-1} - \Phi_7 i_t \\
& + \Phi_8 E_t i_{t+1} - \Phi_9 \hat{b}^R_{t-1} + \Phi_{10} \hat{b}^R_t - \Phi_{11} \hat{g}_{t-1} + \Phi_{12} \hat{g}_t - \Phi_{13} E_t \hat{g}_{t+1} \\
& - \Phi_{14} \tau_t + \Phi_{15} \hat{b}_{t-1} - \Phi_{16} \hat{b}_t + \Phi_{17} \upsilon_t - \Phi_{18} E_t \upsilon_{t+1} - \Phi_5 \upsilon_{t-1} \qquad (3)
\end{aligned}
$$

Inflation and government

- **Firms are rational** which implies the **standard Phillips Curve**

$$\pi_t = \delta(\gamma + \frac{\sigma}{1-\bar{g}})y_t - \delta\sigma(1-\bar{g})^{-1}\hat{g}_t + \beta E_t[\pi_{t+1}] + \mu_t \quad (4)$$

- Government finances government spending with Lump-sum taxes and with government bonds
- Individual bonds follow from budget constraints, bond market clears

Interest rate and shocks

- Monetary policy

$$i_t = \phi_\pi \pi_t + \phi_y y_t \tag{5}$$

- The preference and cost-push shocks

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \epsilon_{\upsilon,t} \tag{6}$$

$$\mu_t = \rho_\mu \mu_{t-1} + \epsilon_{\mu,t} \tag{7}$$

The Rationality Bias

- Focus on differences in the consumption decisions
- **Bias of boundedly rational agents** is

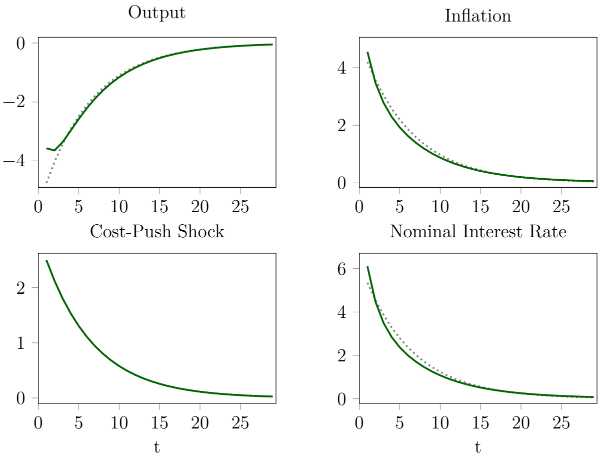
$$\Delta_i c_t^i = c_t^B - c_t^R \tag{8}$$
$$= \underbrace{(E_t^B c_{t+1}^B - E_t c_{t+1}^R)}_{\text{consumption exp. bias}} - \underbrace{\frac{1}{\sigma}(rr_t^B - rr_t)}_{\text{real int. bias}} - \underbrace{\frac{1}{\sigma}(E_t^B \upsilon_{t+1} - E_t \upsilon_{t+1})}_{\text{preference shock exp. bias}}$$

The Rationality Bias

Inserting boundedly rational expectations into the bias gives

$$\Delta_i c_t^i = c_t^B - c_t^R = \underbrace{\left(\frac{1}{1-\bar{g}}\left(y_{t-1} - \hat{g}_{t-1}\right) + \psi(\hat{b}_{t-1}^B - \hat{b}_{t-1}) - E_t c_{t+1}^R\right)}_{\text{consumption exp. bias}} \tag{9}$$

$$\underbrace{+\frac{1}{\sigma}(\pi_{t-1} - E_t \pi_{t+1})}_{\text{neg. real int. bias}} - \underbrace{\frac{1}{\sigma}(\upsilon_{t-1} - E_t \upsilon_{t+1})}_{\text{preference shock exp. bias}} .$$


Output
0
−2
−4
0 5 10 15 20 25
Inflation
4
2
0
0 5 10 15 20 25
Cost-Push Shock
2
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0 5 10 15 20 25
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Nominal Interest Rate
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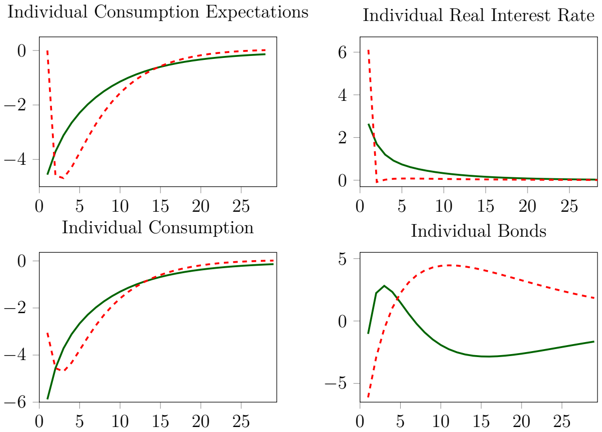
Individual Consumption Expectations
0
−2
−4
0 5 10 15 20 25
Individual Real Interest Rate
6
4
2
0
0 5 10 15 20 25
Individual Consumption
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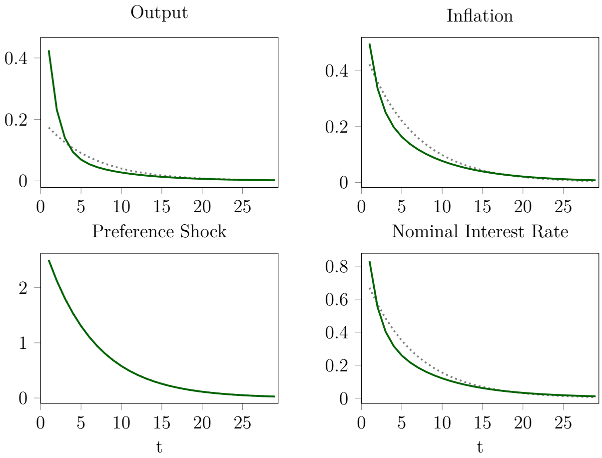
−2
−4
−6
0 5 10 15 20 25
Individual Bonds
5
0
−5
0 5 10 15 20 25


Output
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0.2
0
0 5 10 15 20 25
Inflation
0.4
0.2
0
0 5 10 15 20 25
Preference Shock
2
1
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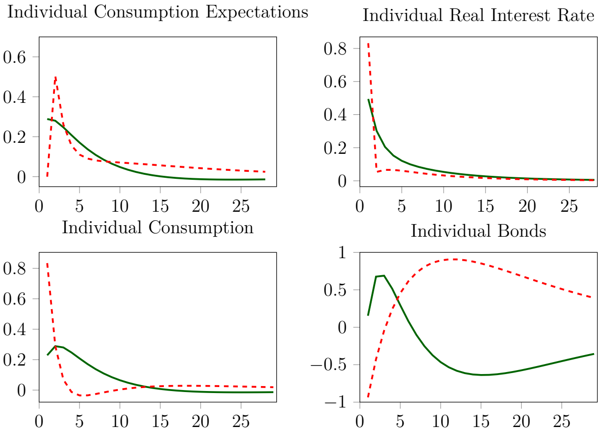
0 5 10 15 20 25
t
Nominal Interest Rate
0.8
0.6
0.4
0.2
0
0 5 10 15 20 25
t


Individual Consumption Expectations
Individual Real Interest Rate
Individual Consumption
Individual Bonds

Eliminating the rationality bias

Proposition

If $\pi_t = i_t$*, then* $c^R_{t-1} = c^B_{t-1}$ *and* $b^B_{t-1} = b_{t-1}$ *imply that* $c^B_{t+s} = c^R_{t+s}$ *and* $b^B_{t+s-1} = b_{t+s-1}$, $s > 0$*. That is, shocks to the economy do not lead to a rationality bias for any parameterization of the model.*

Eliminating the rationality bias: proof

From using $\pi_t = i_t \; \forall t$ and iterating the Euler equation of rational agents until infinity, it follows

$$c_t^R = -\frac{1}{\sigma}\left(\pi_t - \upsilon_t\right). \tag{10}$$

For bounded rational consumption, we use (2). When $\pi_t = i_t$, and assuming $c_{t-1}^R = c_{t-1}^B$ and $b_{t-1}^B = b_{t-1}$, then

$$c_t^B = \frac{1}{1-\bar{g}}(y_{t-1} - \hat{g}_{t-1}) - \frac{1}{\sigma}[\pi_t - \pi_{t-1} - \upsilon_t + \upsilon_{t-1}]. \tag{11}$$

Eliminating the rationality bias: proof

Since it is assumed that $c^B_{t-1} = c^R_{t-1} = -\frac{1}{\sigma}(\pi_{t-1} - \upsilon_{t-1})$, it follows that $y_{t-1} = -\frac{1}{\sigma}(1-\bar{g})(\pi_{t-1} - \upsilon_{t-1}) + \hat{g}_{t-1}$. Therefore, (11) reduces to

$$c^B_t = -\frac{1}{\sigma}(\pi_t - \upsilon_t). \tag{12}$$

This implies that when $c^R_{t-1} = c^B_{t-1}$ and $b^B_{t-1} = b_{t-1}$, it holds that $c^R_t = c^B_t$, so that boundedly rational and rational agents make identical decisions and also $b^B_t = b_t$. This then holds for all periods $s \geq t$, which proves the proposition.

Eliminating the rationality bias: intuition

Using $E_t c^R_{t+1} = -\frac{1}{\sigma} E_t \left(\pi_{t+1} - \upsilon_{t+1}\right)$ and
$y_{t-1} = -\frac{1}{\sigma}(1-\bar{g})\left(\pi_{t-1} - \upsilon_{t-1}\right) + \hat{g}_{t-1}$, reduces the rationality bias to

$$\Delta_i c^i_t = c^B_t - c^R_t = \underbrace{-\frac{1}{\sigma}\Big(\pi_{t-1} - \upsilon_{t-1} - E_t\left(\pi_{t+1} - \upsilon_{t+1}\right)\Big)}_{\text{consumption exp. bias}} \tag{13}$$
$$\underbrace{+\frac{1}{\sigma}(\pi_{t-1} - E_t\pi_{t+1})}_{\text{neg. real int. bias}} - \underbrace{\frac{1}{\sigma}(\upsilon_{t-1} - E_t\upsilon_{t+1})}_{\text{preference shock exp. bias}} = 0.$$

$\Rightarrow$ Biases individually non-zero, but off-set each other exactly

Implementation and Welfare

- Proposition 1 can be implemented by setting the response of the interest rate to inflation (slightly higher than) $\phi_\pi = 1$ and $\phi_y = 0$
- Note, though, that such a weak response of monetary policy implies increased aggregate volatility in the economy
- The second-order approximated aggregate utility loss is given by

$$L_t \simeq a_1 var(y_t) + a_2 var(\pi_t) + a_3 var(c_t^B - c_t^R). \qquad (14)$$

- **It may not be desirable for a central bank to implement a policy where the bias is completely eliminated** since a_2 is considerably bigger than a_3

Conclusion

- **Different degrees of rationality**
- Substantial consumption and wealth heterogeneity can arise when the economy is hit by shocks
- **Rationality bias** of boundedly rational agents is the **driver of consumption and wealth heterogeneity**
- Monetary policy can eliminate this bias
- However, it might not be desirable from a welfare perspective to completely eliminate the rationality bias