Government Debt Maturity Structure, Fiscal Policy, and Default

Sergii Kiiashko

Kyiv School of Economics and National Bank of Ukraine

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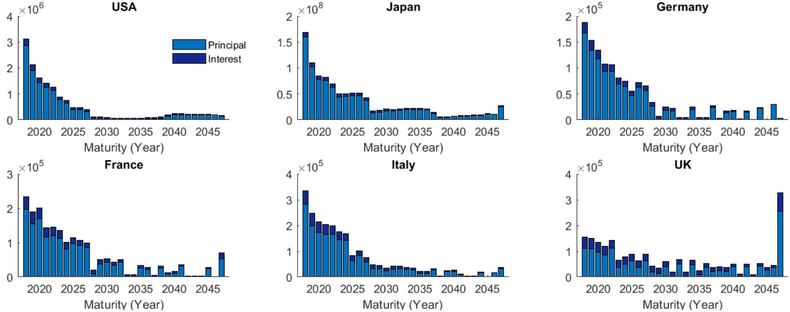
Government Debt Maturity Structure



Source: Bloomberg. Data collected on 17th of October, 2017.

Introduction

Motivation:

- **Empirical facts**: decaying maturity profile
- **No consensus** on how debt maturity should be managed: more short-term debt or long-term debt?
- **No theory** that explains the decaying profile of maturity:
  - Lucas and Stokey (1983): endogenous risk-free interest rates but no default $\Rightarrow$ **flat maturity structure**
  - Aguiar et al (2018): endogenous default but exogenous risk-free rates $\Rightarrow$ **issue only one-period debt**

Questions:

1. What is the **optimal government debt maturity structure** in an environment with lack of commitment and opportunity to default, if **both risk-free interest rates and risk premiums** are endogenous?
2. Are predictions of this model consistent with empirical observations?

This Project

A la Lucas and Stokey (1983) **model**:

- closed economy: government borrows from households
- lack of commitment
- strictly concave utility over consumption

  $\Rightarrow$ **risk-free interest rates** are endogenous

With opportunity to default as in Aguiar, Amador, Hopenhayn, and Werning (2018):

- default risk is continuously increasing in outstanding debt

  $\Rightarrow$ **risk premiums** are endogenous

Results:

1. **Solution is time-consistent**: government follows ex ante optimal fiscal policies conditional on no default.
2. In the presence of sufficient default risk, **the optimal maturity structure has a decaying profile.**

Outline

A Three-Period Example

- Three periods
- A governemnt is endowed with τ every period and maximizes $\theta_0\omega(g_0) + \omega(g_1) + \omega(g_2)$
- No initial debt but $\theta_0 > 1$ so there is incentive to borrow
- Government borrows from lenders $(b_0^1,\ b_0^2,\ b_1^2)$
- Important features:
  - lack of commitment
  - risk-averse lenders with $u(c_t)$, $c_t + g_t = 1$
  - default risk in period 2 with $\pi'(b_1^2) \geq 0$

Modified Commitment Problem

Consider a planner who can commit to policy in period 1.

To simplify notation, let $s_t = \tau - g_t,\ c_t = 1 - \tau + s_t,\ b_1^2 = s_2$

Optimization problem is

$$\max_{s_0,\, s_1,\, s_2} \theta_0 \omega(\tau - s_0) + \omega(\tau - s_1) + \mathbb{E} V_2(\tau - s_2)$$

$$\text{s.t. } s_0 + \frac{u'(1-\tau+s_1)}{u'(1-\tau+s_0)} s_1 + \frac{u'(1-\tau+s_2)}{u'(1-\tau+s_0)}(1-\pi(s_2))s_2 = 0$$

The maturity structure is **irrelevant**.

Let $(s_0^\star,\, s_1^\star,\, s_2^\star)$ denote the optimal policy. Can the government with lack of commitment achieve the planner's allocation?

Optimal Maturity Structure

Case of **commitment**:

$$\max_{s_1,\, s_2} \omega(\tau - s_1) + \mathbb{E} V_2(\tau - s_2)$$

$$\text{s.t. } u'(1-\tau+s_1)s_1 + u'(1-\tau+s_2)(1-\pi(s_2))s_2 = -u'(1-\tau+s_0)s_0$$

Case of **no commitment** (same utility function but different budget constraint):

$$\text{s.t. } u'(1-\tau+s_1)s_1 + u'(1-\tau+s_2)(1-\pi(s_2))s_2 = u'(1-\tau+s_1)b_0^1 + u'(1-\tau+s_2)(1-\pi(s_2))b_0^2$$

Optimal maturity structure of debt should satisfy:

$$\frac{\partial u'(1-\tau+s_1^\star)}{\partial s_1}\triangle s_1 \cdot b_0^1 + \frac{\partial u'(1-\tau+s_2^\star)(1-\pi(s_2^\star))}{\partial s_2}\triangle s_2 \cdot b_0^2 = 0$$

$$\Rightarrow \frac{b_0^1}{b_0^2} = \frac{\frac{\partial u'(1-\tau+s_2^\star)(1-\pi(s_2^\star))}{\partial s_2}}{\frac{\partial u'(1-\tau+s_1^\star)}{\partial s_1}} \cdot MRS^\star$$

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Discussion

Lucas and Stokey, 1983:

suppose $\pi = \pi' = 0 \Rightarrow s_1^\star = s_2^\star \Rightarrow b_0^1 = b_0^2$

if maturity is not flat, say $b_0^1 > b_0^2$, government has incentive to increase s_1 and decrease s_2.

Aguiar et al., 2018:

suppose $u' = const \Rightarrow b_0^2 = 0$

if $b_2 > 0$, government has incentive to increase s_2 and decrease s_1

This model:

ST debt is manipulated by changes in risk-free interest rate

LT debt is manipulated by changes in rf interest rate AND default premium

$\Rightarrow$ generally, once government changes surplus, LT interest rates respond stronger than ST $\Rightarrow$ $b_0^1 > b_0^2 > 0$

Overview of the Model

Representative household:

- values private consumption c_t and government spending g_t:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(u(c_t)+\theta_t\omega(g_t)\right)$$

- endowed with 1 unit of consumption
- optimally chooses consumption and savings (government bonds)

Government:

- benevolent (same preference)
- collects τ in taxes
- optimally chooses government spending and issues bonds b_t^{t+k}, q_t^{t+k} is price
- lacks commitment (makes decisions every period) and can default
- if defaults receives V_t^{def} which is stochastic and continuously distributed

Resource constraint: $c_t + g_t \leq 1\,\forall t.$

Assumptions: $\theta_0 > \theta_1 = \theta_2 = \ldots = \theta_T = 1$, $\omega'(\tau) \geq u'(1-\tau)$.

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Markov Perfect Competitive Equilibrium

State variables: $\boldsymbol{b}_{t-1} = (b_{t-1}^{t},\ b_{t-1}^{t+1},\ \ldots)$ and V_t^{def}

Timing: 1. Default decision 2. Fiscal and Debt Decisions

Primary budget surplus: $s_t = \tau - g_t$

$$V_t(\boldsymbol{b}_{t-1}) = \max_{s_t,\, \boldsymbol{b}_t} \left\{ u(1-\tau+s_t) + \theta_t \omega(\tau - s_t) + \beta \cdot \mathbb{E} \max \left\{ V_{t+1}(\boldsymbol{b}_t);\ V_{t+1}^{def} \right\} \right\}$$

$$\text{s.t.} \quad \underbrace{s_t + \boldsymbol{q}_t(s_t,\, \boldsymbol{b}_t) \cdot \boldsymbol{b}_t}_{\text{budget surplus and issued debt}} \geq \underbrace{(1,\ \boldsymbol{q}_t(s_t,\, \boldsymbol{b}_t)) \cdot \boldsymbol{b}_{t-1}}_{\text{outstanding debt}}$$

$$\underbrace{q_t^{t+k}(s_t,\, \boldsymbol{b}_t)}_{\text{price of bond maturing at date } t+k} = \underbrace{\beta \frac{u'(1-\tau+s_{t+1}^{\star}(\boldsymbol{b}_t))}{u'(1-\tau+s_t)}}_{\text{risk-free part}} \cdot \underbrace{(1-\pi_{t+1}(\boldsymbol{b}_t))}_{\text{default premium part}} \cdot \underbrace{q_{t+1}^{t+k}\left(s_{t+1}^{\star}(\boldsymbol{b}_t),\ \boldsymbol{b}_{t+1}^{\star}(\boldsymbol{b}_t)\right)}_{\text{price of bond maturing at date } t+k \text{ tomorrow}}$$

where $\pi_{t+1}(\boldsymbol{b}_t) = \text{Prob}(V_{t+1}^{def} > V_{t+1}(\boldsymbol{b}_t))$ is **default risk**.

MPCE: value $V_t(\boldsymbol{b}_{t-1})$, policy $s_t^{\star}(\boldsymbol{b}_{t-1})$, $\boldsymbol{b}_t^{\star}(\boldsymbol{b}_{t-1})$, and pricing functions $\boldsymbol{q}_t(s_t,\, \boldsymbol{b}_t)$.

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Modified Commitment Problem

Planner commits to fiscal policies, but cannot commit to repay debt.

Fiscal plan $\boldsymbol{s}_0 = (s_0,\ s_1,\ ...,\ s_t,\ ...)$: a sequence of contingent budget surpluses.

Value of fiscal plan at t is defined recursively

$$W_t(\boldsymbol{s}_t) = u(1-\tau+s_t) + \theta_t\omega(\tau-s_t) + \beta\cdot\mathbb{E}\max\left\{W_{t+1}(\boldsymbol{s}_{t+1}\in\boldsymbol{s}_t),\ V^{def}_{t+1}\right\}$$

Planner's problem in period T is **to maximize utility subject to the dynamic budget constraint:**

$$\max_{\{\boldsymbol{s}_T\}} W_T(\boldsymbol{s}_T)$$

$$\text{s.t.}\ \underbrace{\sum_{k=T}^{\infty}\beta^{k-T}u'_k\cdot Pr_T^{T+k}\cdot s_k}_{\equiv S_T} \geq \underbrace{\sum_{k=0}^{\infty}\beta^{k-T}u'_k\cdot Pr_T^{T+k}\cdot b_{T-1}^{T+k}}_{\equiv B_{T-1,\ T}}$$

where $Pr_T^{T+k}(\boldsymbol{s}_{T+1}) = \prod_{t=T+1}^{T+k}\text{Prob}(W_t(\boldsymbol{s}_t)\geq V_t^{def})$ - probability of repaying bond with maturity k issued in period T.

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Optimal Maturity Structure

FOC of planner at period 0 :

$$\frac{\theta_{t+1}\omega'_{t+1} - u'_{t+1}}{\theta_t\omega'_t - u'_t} = \frac{\frac{\partial\left(u'_{t+1}\cdot(s_{t+1}-b^{t+1}_{-1})\right)}{\partial s_{t+1}} + \frac{\frac{\partial Pr^{t+1}_t}{\partial s_{t+1}}}{Pr^{t+1}_t}\left(S_{t+1} - B_{-1,\,t+1}\right)}{\frac{\partial\left(u'_t\cdot(s_t-b^t_{-1})\right)}{\partial s_t}}$$

FOC of planner at period $T \leq t$**:**

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Optimal maturity structure:

$$\frac{\theta_{t+1}\hat{\omega}'_{t+1} - \hat{u}'_{t+1}}{\theta_t\hat{\omega}'_t - \hat{u}'_t} = \frac{\frac{\partial\left(\hat{u}'_{t+1}\cdot(b^{t+1}_{-1}-b^{t+1}_{T-1})\right)}{\partial s_{t+1}} + \frac{\frac{\partial \hat{P}r^{t+1}_t}{\partial s_{t+1}}}{\hat{P}r^{t+1}_t}\left(B_{-1,\,t+1} - B_{T-1,\,t+1}\right)}{\frac{\partial\left(\hat{u}'_t\cdot(b^t_{-1}-b^t_{T-1})\right)}{\partial s_t}}$$

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Discussion

Maturity structure serves as a commitment device: no deviations from ex ante optimal fiscal policy can decrease market value of outstanding debt.

Perturbation: marginal increase s_t and decrease s_{t+1}, $t \geq 1$, keeping $W_t(\boldsymbol{s}_t)$

Risk-free interest rates: $u'(1 - \tau + s_k)$ changes for $k = t,\ t+1$

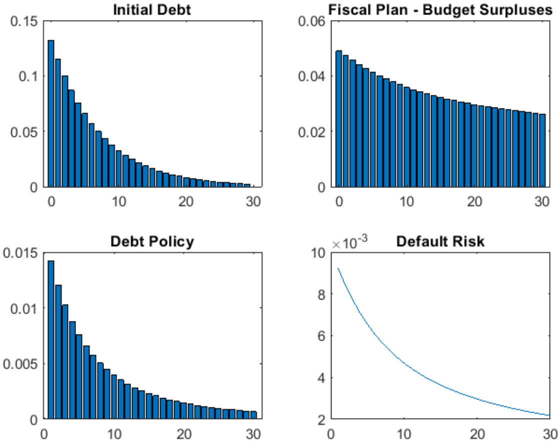
Default premium: default risk at $t+1$ changes due to change in $W_{t+1}(\boldsymbol{s}_{t+1})$,

but default risk at t does not change because $W_t(\boldsymbol{s}_t)$ is constant

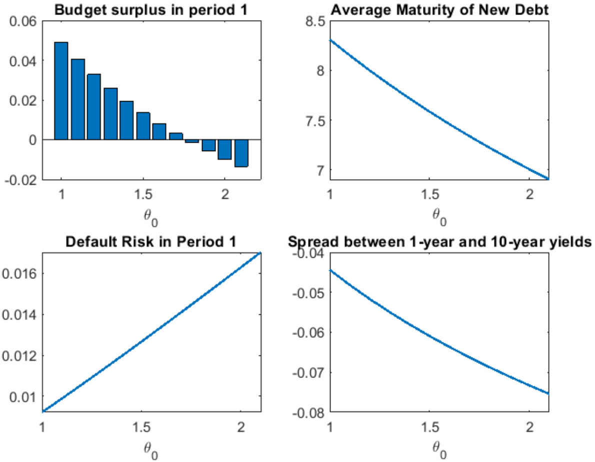
Assymetry: long-term interest rates are more elastic than short-term

Proposition: under certain reasonable assumptions we can analytically prove that $b^T_{T-1} > b^{T+1}_{T-1} > ... > b^{T+k}_{T-1} > ... > 0$.

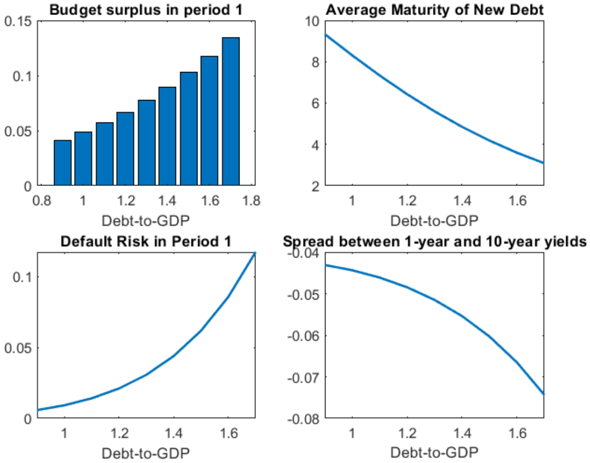
Benchmark Case



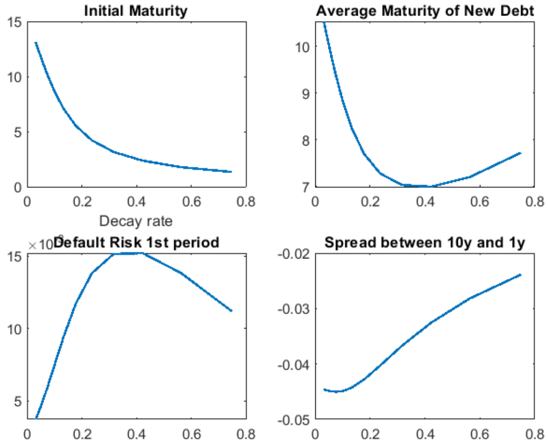
Shock to Taste Parameter



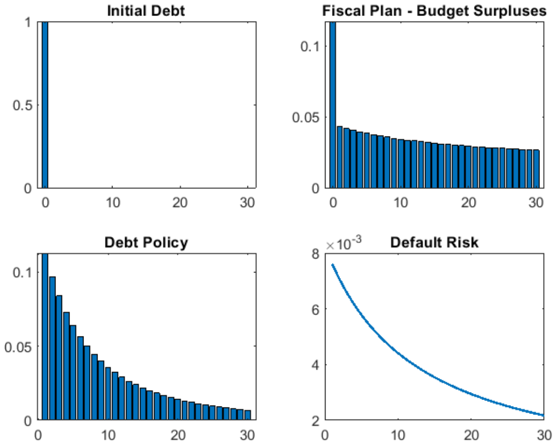
Changing Debt-to-GDP



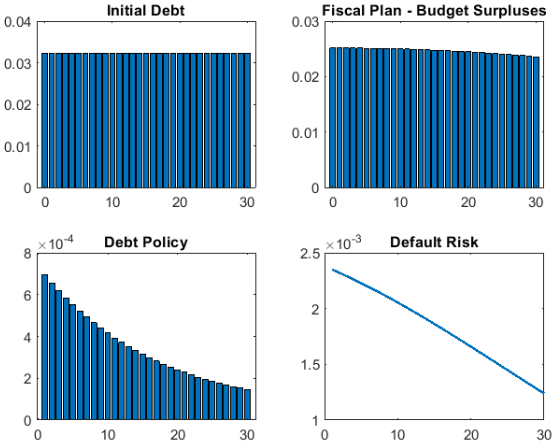
Changing Decaying Rate of Maturity Structure



Short Initial Debt



Long Initial Debt



Summary

Main results:

- Maturity is used to discipline the future governments
- Asymmetry: longer-term interest rates are more elastic than shorter-term rates
- Generally, maturity structure has a decaying profile
- Numerical exercises can be performed to study role of initial maturity structure, debt-to-GDP ratio, unexpected shock to risk premium etc.