# Does too much finance generate instability?

G. Calcagnini L. Gardini G. Giombini and E. S-Carrera

Department of Economics, Society and Politics
Università di Urbino Carlo Bo
61029 Urbino, Italy

*11th Nonlinear Economic Dynamics Conference 2019*

Kiev, 4 - 6 September, 2019

# Motivation

- Corporate demand for cash is related to a number of firm-specific characteristics, like the presence of transaction costs, information asymmetry in credit markets, uncertainty and risk aversion.
- Indeed, in the presence of imperfect capital markets, the benefit for firms of holding cash is the cost avoidance associated with the external-fund raising or the liquidation of existing assets to finance their growth opportunities (Calcagnini, Gehr, Giombini, 2009).
- For example, when agency problems exist, i.e., when the interests of controlling shareholders are not aligned with those of outside investors, controlling shareholders prefer to keep funds in liquid assets that have a private benefit option attached to them that other assets do not have.

# Motivation

- Corporate demand for cash is related to a number of firm-specific characteristics, like the presence of transaction costs, information asymmetry in credit markets, uncertainty and risk aversion.
- Indeed, in the presence of imperfect capital markets, the benefit for firms of holding cash is the cost avoidance associated with the external-fund raising or the liquidation of existing assets to finance their growth opportunities (Calcagnini, Gehr, Giombini, 2009).
- For example, when agency problems exist, i.e., when the interests of controlling shareholders are not aligned with those of outside investors, controlling shareholders prefer to keep funds in liquid assets that have a private benefit option attached to them that other assets do not have.

# Motivation

- Corporate demand for cash is related to a number of firm-specific characteristics, like the presence of transaction costs, information asymmetry in credit markets, uncertainty and risk aversion.
- Indeed, in the presence of imperfect capital markets, the benefit for firms of holding cash is the cost avoidance associated with the external-fund raising or the liquidation of existing assets to finance their growth opportunities (Calcagnini, Gehr, Giombini, 2009).
- For example, when agency problems exist, i.e., when the interests of controlling shareholders are not aligned with those of outside investors, controlling shareholders prefer to keep funds in liquid assets that have a private benefit option attached to them that other assets do not have.

# Motivation

- Furthermore, liquidity is an investment in short term assets alternative to other forms of resource allocation with important implications for corporate profitability, risk and financial soundness, and, more generally, for economic growth;
- on the other hand, cash holding costs are mainly the opportunity cost of cash, i.e., the lower return of liquid assets relative to other investments of the same level of risk (Dottori and Micucci, 2018).
- In this paper we explore a theoretical connection between firm financial policies, investment decisions, and chaotic time paths, showing that too much finance might generate instability.

# Motivation

- Furthermore, liquidity is an investment in short term assets alternative to other forms of resource allocation with important implications for corporate profitability, risk and financial soundness, and, more generally, for economic growth;
- on the other hand, cash holding costs are mainly the opportunity cost of cash, i.e., the lower return of liquid assets relative to other investments of the same level of risk (Dottori and Micucci, 2018).
- In this paper we explore a theoretical connection between firm financial policies, investment decisions, and chaotic time paths, showing that too much finance might generate instability.

# Motivation

- Furthermore, liquidity is an investment in short term assets alternative to other forms of resource allocation with important implications for corporate profitability, risk and financial soundness, and, more generally, for economic growth;
- on the other hand, cash holding costs are mainly the opportunity cost of cash, i.e., the lower return of liquid assets relative to other investments of the same level of risk (Dottori and Micucci, 2018).
- In this paper we explore a theoretical connection between firm financial policies, investment decisions, and chaotic time paths, showing that too much finance might generate instability.

# Introduction

- Data for large firms show that there is an excess of liquidity, which generates signals that are not perceived correctly by the market. ▸ MedioBanca

Table: Resources and Uses of 2095 Italian Firms (millions of euro).

| | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|
| Technical Investment | 28,619 | 28,445 | 29,402 | 29,579 |
| Financial Investment | 17,432 | 23,542 | 15,874 | 25,199 |
| Liquidity variations | 4,140 | 1,692 | 8,823 | 1,753 |
| Working capital variations | -3,894 | -11,022 | -2,099 | 645 |
| Total uses | 46,297 | 42,657 | 52,000 | 57,176 |
| Cash-flow | 59,704 | 58,134 | 62,412 | 63,290 |
| Contributions from shareholders | -10,332 | -1,473 | -17,602 | -18,035 |
| Other contributions | - | - | - | - |
| Financial Debt variations | -3,075 | -14,004 | 7,190 | 11,921 |
| Total resources | 46,297 | 42,657 | 52,000 | 57,176 |

**Source**: MedioBanca (2019). Dati Cumulativi di 2095 Società Italiane.

- Data for large firms show that there is an excess of liquidity, which generates signals that are not perceived correctly by the market. MedioBanca

Table: Resources and Uses of 2095 Italian Firms (millions of euro).

| | 2015 | 2016 | 2017 | 2018 |
|---|---|---|---|---|
| Technical Investment | 28,619 | 28,445 | 29,402 | 29,579 |
| Financial Investment | 17,432 | 23,542 | 15,874 | 25,199 |
| Liquidity variations | 4,140 | 1,692 | 8,823 | 1,753 |
| Working capital variations | -3,894 | -11,022 | -2,099 | 645 |
| Total uses | 46,297 | 42,657 | 52,000 | 57,176 |
| Cash-flow | 59,704 | 58,134 | 62,412 | 63,290 |
| Contributions from shareholders | -10,332 | -1,473 | -17,602 | -18,035 |
| Other contributions | - | - | - | - |
| Financial Debt variations | -3,075 | -14,004 | 7,190 | 11,921 |
| Total resources | 46,297 | 42,657 | 52,000 | 57,176 |

**Source**: MedioBanca (2019). Dati Cumulativi di 2095 Società Italiane.

# Introduction

- Moreover, main central banks are currently engaged in a new round of monetary expansion, with the aim to reduce interest rates to strengthen the economic cycle and, in the case of the ECB, try to achieve the inflation target.
- A possible (negative) consequence is that the liquidity introduced by the central banks in the financial markets could be used in the search for returns to generate and feed a speculative bubble in some assets (real estate, securities prices traded on the exchange, etc.) (Monticini, 2019)
- If a speculative bubble were to form in some assets, the subsequent outbreak could have negative consequences on the real economy.

# Introduction

- Moreover, main central banks are currently engaged in a new round of monetary expansion, with the aim to reduce interest rates to strengthen the economic cycle and, in the case of the ECB, try to achieve the inflation target.
- A possible (negative) consequence is that the liquidity introduced by the central banks in the financial markets could be used in the search for returns to generate and feed a speculative bubble in some assets (real estate, securities prices traded on the exchange, etc.) (Monticini, 2019)
- If a speculative bubble were to form in some assets, the subsequent outbreak could have negative consequences on the real economy.

# Introduction

- Moreover, main central banks are currently engaged in a new round of monetary expansion, with the aim to reduce interest rates to strengthen the economic cycle and, in the case of the ECB, try to achieve the inflation target.
- A possible (negative) consequence is that the liquidity introduced by the central banks in the financial markets could be used in the search for returns to generate and feed a speculative bubble in some assets (real estate, securities prices traded on the exchange, etc.) (Monticini, 2019)
- If a speculative bubble were to form in some assets, the subsequent outbreak could have negative consequences on the real economy.

# Introduction

- What do large firms do with this excess liquidity?
- Large companies tend to invest excess liquidity in financial assets
- In this context, the distribution of the firm dividend, coming from firm financial activities more than from the firm production, provides a wrong signal to the markets.
- Thus, dividend changes are not intended to signal future earnings. Instead, firms change the dividend to return excess cash to shareholders as in standard capital structure models (Myers and Majluf 1984).
- The return of cash often triggers large market reactions because excess cash is a function of earnings, and thus reveals the earnings information to investors (Kaplan and Perez-Cavazos, 2019).
- Finally, the instability of financial markets ends up generating costs both in real and in financial markets.

# Introduction

- What do large firms do with this excess liquidity?
- Large companies tend to invest excess liquidity in financial assets
- In this context, the distribution of the firm dividend, coming from firm financial activities more than from the firm production, provides a wrong signal to the markets.
- Thus, dividend changes are not intended to signal future earnings. Instead, firms change the dividend to return excess cash to shareholders as in standard capital structure models (Myers and Majluf 1984).
- The return of cash often triggers large market reactions because excess cash is a function of earnings, and thus reveals the earnings information to investors (Kaplan and Perez-Cavazos, 2019).
- Finally, the instability of financial markets ends up generating costs both in real and in financial markets.

# Introduction

- What do large firms do with this excess liquidity?
- Large companies tend to invest excess liquidity in financial assets
- In this context, the distribution of the firm dividend, coming from firm financial activities more than from the firm production, provides a wrong signal to the markets.
- Thus, dividend changes are not intended to signal future earnings. Instead, firms change the dividend to return excess cash to shareholders as in standard capital structure models (Myers and Majluf 1984).
- The return of cash often triggers large market reactions because excess cash is a function of earnings, and thus reveals the earnings information to investors (Kaplan and Perez-Cavazos, 2019).
- Finally, the instability of financial markets ends up generating costs both in real and in financial markets.

# Introduction

- What do large firms do with this excess liquidity?
- Large companies tend to invest excess liquidity in financial assets
- In this context, the distribution of the firm dividend, coming from firm financial activities more than from the firm production, provides a wrong signal to the markets.
- Thus, dividend changes are not intended to signal future earnings. Instead, firms change the dividend to return excess cash to shareholders as in standard capital structure models (Myers and Majluf 1984).
- The return of cash often triggers large market reactions because excess cash is a function of earnings, and thus reveals the earnings information to investors (Kaplan and Perez-Cavazos, 2019).
- Finally, the instability of financial markets ends up generating costs both in real and in financial markets.

# Introduction

- What do large firms do with this excess liquidity?
- Large companies tend to invest excess liquidity in financial assets
- In this context, the distribution of the firm dividend, coming from firm financial activities more than from the firm production, provides a wrong signal to the markets.
- Thus, dividend changes are not intended to signal future earnings. Instead, firms change the dividend to return excess cash to shareholders as in standard capital structure models (Myers and Majluf 1984).
- The return of cash often triggers large market reactions because excess cash is a function of earnings, and thus reveals the earnings information to investors (Kaplan and Perez-Cavazos, 2019).
- Finally, the instability of financial markets ends up generating costs both in real and in financial markets.

## Introduction

- What do large firms do with this excess liquidity?
- Large companies tend to invest excess liquidity in financial assets
- In this context, the distribution of the firm dividend, coming from firm financial activities more than from the firm production, provides a wrong signal to the markets.
- Thus, dividend changes are not intended to signal future earnings. Instead, firms change the dividend to return excess cash to shareholders as in standard capital structure models (Myers and Majluf 1984).
- The return of cash often triggers large market reactions because excess cash is a function of earnings, and thus reveals the earnings information to investors (Kaplan and Perez-Cavazos, 2019).
- Finally, the instability of financial markets ends up generating costs both in real and in financial markets.

## The model

- Firms hold cash ($\Pi$) that is used to pay dividend $D$, to finance investment projects ($I$) and to buy bonds ($B$). Firms may finance its real and financial investment decisions by borrowing funds ($L$) from financial markets.
- Thus, the following equation relates the beginning of period cash $\Pi_{t-1}$ to D, I, B and L as follows

$$\Pi_{t-1} = D_t + I_t + \Delta \tilde{B}_t$$

where $\tilde{B}_t$ is net bonds, that is the difference between bought bonds and borrowed loans from the financial markets, $\tilde{B}_t = B_t - L_t$ so that $\Delta \tilde{B}_t = \Delta B_t - \Delta L_t$
- As corporate managers tend to upload the concept of a stable long-term payout ratio (Shaffer, 1991), we assume that firms pay dividend D as a fixed proportion of beginning of period cash

$$D_t = \alpha_1 \Pi_{t-1}$$

where $\alpha_1$ is constant.

## The model

- Firms hold cash ($\Pi$) that is used to pay dividend $D$, to finance investment projects ($I$) and to buy bonds ($B$). Firms may finance its real and financial investment decisions by borrowing funds ($L$) from financial markets.
- Thus, the following equation relates the beginning of period cash $\Pi_{t-1}$ to D, I, B and L as follows

$$\Pi_{t-1} = D_t + I_t + \Delta\tilde{B}_t$$

where $\tilde{B}_t$ is net bonds, that is the difference between bought bonds and borrowed loans from the financial markets, $\tilde{B}_t = B_t - L_t$ so that $\Delta\tilde{B}_t = \Delta B_t - \Delta L_t$

- As corporate managers tend to upload the concept of a stable long-term payout ratio (Shaffer, 1991), we assume that firms pay dividend D as a fixed proportion of beginning of period cash

$$D_t = \alpha_1 \Pi_{t-1}$$

where $\alpha_1$ is constant.

# The model

- Firms hold cash ($\Pi$) that is used to pay dividend $D$, to finance investment projects ($I$) and to buy bonds ($B$). Firms may finance its real and financial investment decisions by borrowing funds ($L$) from financial markets.
- Thus, the following equation relates the beginning of period cash $\Pi_{t-1}$ to D, I, B and L as follows

$$\Pi_{t-1} = D_t + I_t + \Delta \tilde{B}_t$$

  where $\tilde{B}_t$ is net bonds, that is the difference between bought bonds and borrowed loans from the financial markets, $\tilde{B}_t = B_t - L_t$ so that $\Delta \tilde{B}_t = \Delta B_t - \Delta L_t$
- As corporate managers tend to upload the concept of a stable long-term payout ratio (Shaffer, 1991), we assume that firms pay dividend D as a fixed proportion of beginning of period cash

$$D_t = \alpha_1 \Pi_{t-1}$$

  where $\alpha_1$ is constant.

# The model

- Firms hold cash ($\Pi$) that is used to pay dividend $D$, to finance investment projects ($I$) and to buy bonds ($B$). Firms may finance its real and financial investment decisions by borrowing funds ($L$) from financial markets.
- Thus, the following equation relates the beginning of period cash $\Pi_{t-1}$ to D, I, B and L as follows

$$\Pi_{t-1} = D_t + I_t + \Delta\tilde{B}_t$$

  where $\tilde{B}_t$ is net bonds, that is the difference between bought bonds and borrowed loans from the financial markets, $\tilde{B}_t = B_t - L_t$ so that $\Delta\tilde{B}_t = \Delta B_t - \Delta L_t$
- As corporate managers tend to upload the concept of a stable long-term payout ratio (Shaffer, 1991), we assume that firms pay dividend D as a fixed proportion of beginning of period cash

$$D_t = \alpha_1 \Pi_{t-1}$$

where $\alpha_1$ is constant.

# The model

- Firms hold cash ($\Pi$) that is used to pay dividend $D$, to finance investment projects ($I$) and to buy bonds ($B$). Firms may finance its real and financial investment decisions by borrowing funds ($L$) from financial markets.
- Thus, the following equation relates the beginning of period cash $\Pi_{t-1}$ to D, I, B and L as follows

$$\Pi_{t-1} = D_t + I_t + \Delta \tilde{B}_t$$

where $\tilde{B}_t$ is net bonds, that is the difference between bought bonds and borrowed loans from the financial markets, $\tilde{B}_t = B_t - L_t$ so that $\Delta \tilde{B}_t = \Delta B_t - \Delta L_t$

- As corporate managers tend to upload the concept of a stable long-term payout ratio (Shaffer, 1991), we assume that firms pay dividend D as a fixed proportion of beginning of period cash

$$D_t = \alpha_1 \Pi_{t-1}$$

where $\alpha_1$ is constant.

# The model

- The remainder of beginning of period cash is reinvested either in I or used to buy bonds B.
- Net bonds have a rate of return equal to $i$, which can be interpreted as the spread between the interest rate on bonds minus interest rate on loans
- We assume that following an increase in interest rates, firms find more convenient to buy bonds (and to borrow less)
- This assumption is modeled as follows

$$I_t = \alpha_2 \Pi_{t-1}$$

where $\alpha_2(i_t)$ so that $\partial \alpha_2 / \partial i_t \leq\geq 0$.

$$\Delta \tilde{B}_t = \alpha_3 \Pi_{t-1}$$

where $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and $\partial \alpha_3 / \partial i_t > 0$

# The model

- The remainder of beginning of period cash is reinvested either in I or used to buy bonds B.
- Net bonds have a rate of return equal to $i$, which can be interpreted as the spread between the interest rate on bonds minus interest rate on loans
- We assume that following an increase in interest rates, firms find more convenient to buy bonds (and to borrow less)
- This assumption is modeled as follows

$$I_t = \alpha_2 \Pi_{t-1}$$

where $\alpha_2(i_t)$ so that $\partial \alpha_2 / \partial i_t \leq \geq 0$.

$$\Delta \tilde{B}_t = \alpha_3 \Pi_{t-1}$$

where $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and $\partial \alpha_3 / \partial i_t > 0$

- The remainder of beginning of period cash is reinvested either in I or used to buy bonds B.
- Net bonds have a rate of return equal to $i$, which can be interpreted as the spread between the interest rate on bonds minus interest rate on loans
- We assume that following an increase in interest rates, firms find more convenient to buy bonds (and to borrow less)
- This assumption is modeled as follows

$$I_t = \alpha_2 \Pi_{t-1}$$

where $\alpha_2(i_t)$ so that $\partial \alpha_2 / \partial i_t \leq \geq 0$.

$$\Delta \tilde{B}_t = \alpha_3 \Pi_{t-1}$$

where $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and $\partial \alpha_3 / \partial i_t > 0$

# The model

- The remainder of beginning of period cash is reinvested either in I or used to buy bonds B.
- Net bonds have a rate of return equal to $i$, which can be interpreted as the spread between the interest rate on bonds minus interest rate on loans
- We assume that following an increase in interest rates, firms find more convenient to buy bonds (and to borrow less)
- This assumption is modeled as follows

$$I_t = \alpha_2 \Pi_{t-1}$$

where $\alpha_2(i_t)$ so that $\partial \alpha_2 / \partial i_t \leq \geq 0$.

$$\Delta \tilde{B}_t = \alpha_3 \Pi_{t-1}$$

where $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and $\partial \alpha_3 / \partial i_t > 0$

# Cash holdings

- We assume that the end of the period cash holdings $\Pi_t$ depends on:
  1. the beginning of period cash $\Pi_{t-1}$
  2. total earnings from new investment projects $R_t$,
  3. net bond yields $Y_t$,
  4. the portion $(1-\rho)$ of the previous-period net securities that expired and has not been renewed
- Thus, firm cash holdings evolve according to the growth path defined as follows

$$\Pi_t = \Pi_{t-1} + R_t + Y_t + (1-\rho)\tilde{B}_{t-1}$$

## Cash holdings

- We assume that the end of the period cash holdings $\Pi_t$ depends on:
  1. the beginning of period cash $\Pi_{t-1}$
  2. total earnings from new investment projects $R_t$,
  3. net bond yields $Y_t$,
  4. the portion $(1-\rho)$ of the previous-period net securities that expired and has not been renewed
- Thus, firm cash holdings evolve according to the growth path defined as follows

$$\Pi_t = \Pi_{t-1} + R_t + Y_t + (1-\rho)\tilde{B}_{t-1}$$

# Cash holdings

- We assume that the end of the period cash holdings $\Pi_t$ depends on:
  1. the beginning of period cash $\Pi_{t-1}$
  2. total earnings from new investment projects $R_t$,
  3. net bond yields $Y_t$,
  4. the portion $(1-\rho)$ of the previous-period net securities that expired and has not been renewed
- Thus, firm cash holdings evolve according to the growth path defined as follows

$$\Pi_t = \Pi_{t-1} + R_t + Y_t + (1-\rho)\tilde{B}_{t-1}$$

# Cash holdings

- We assume that the end of the period cash holdings $\Pi_t$ depends on:
  1. the beginning of period cash $\Pi_{t-1}$
  2. total earnings from new investment projects $R_t$,
  3. net bond yields $Y_t$,
  4. the portion $(1-\rho)$ of the previous-period net securities that expired and has not been renewed
- Thus, firm cash holdings evolve according to the growth path defined as follows

$$\Pi_t = \Pi_{t-1} + R_t + Y_t + (1-\rho)\tilde{B}_{t-1}$$

# Cash holdings

- We assume that the end of the period cash holdings $\Pi_t$ depends on:
  1. the beginning of period cash $\Pi_{t-1}$
  2. total earnings from new investment projects $R_t$,
  3. net bond yields $Y_t$,
  4. the portion $(1-\rho)$ of the previous-period net securities that expired and has not been renewed
- Thus, firm cash holdings evolve according to the growth path defined as follows

$$\Pi_t = \Pi_{t-1} + R_t + Y_t + (1-\rho)\tilde{B}_{t-1}$$

# Cash holdings

- We assume that the end of the period cash holdings $\Pi_t$ depends on:
  1. the beginning of period cash $\Pi_{t-1}$
  2. total earnings from new investment projects $R_t$,
  3. net bond yields $Y_t$,
  4. the portion $(1-\rho)$ of the previous-period net securities that expired and has not been renewed
- Thus, firm cash holdings evolve according to the growth path defined as follows

$$\Pi_t = \Pi_{t-1} + R_t + Y_t + (1-\rho)\tilde{B}_{t-1}$$

# Marginal efficiency of investment

- We assume a marginal efficiency of investment (MEI) curve that declines linearly down to some minimum rate of return, such as a risk free rate plus an appropriate risk premium.
- Indeed, reliance on external finance for some portion of investment could lead to a downward sloping net MEI curve, if external financing is more costly and if it is used for successively greater levels of investments (Shaffer 1991).
- Thus, the investment total return is defined according to the following equation

$$R_t = aI_t - b\frac{I_t^2}{2}$$

- so that the investment earns a marginal return of

$$\frac{dR_t}{dI_t} = a - bI_t$$

# Marginal efficiency of investment

- We assume a marginal efficiency of investment (MEI) curve that declines linearly down to some minimum rate of return, such as a risk free rate plus an appropriate risk premium.
- Indeed, reliance on external finance for some portion of investment could lead to a downward sloping net MEI curve, if external financing is more costly and if it is used for successively greater levels of investments (Shaffer 1991).
- Thus, the investment total return is defined according to the following equation

$$R_t = aI_t - b\frac{I_t^2}{2}$$

- so that the investment earns a marginal return of

$$\frac{dR_t}{dI_t} = a - bI_t$$

# Marginal efficiency of investment

- We assume a marginal efficiency of investment (MEI) curve that declines linearly down to some minimum rate of return, such as a risk free rate plus an appropriate risk premium.
- Indeed, reliance on external finance for some portion of investment could lead to a downward sloping net MEI curve, if external financing is more costly and if it is used for successively greater levels of investments (Shaffer 1991).
- Thus, the investment total return is defined according to the following equation

$$R_t = aI_t - b\frac{I_t^2}{2}$$

- so that the investment earns a marginal return of

$$\frac{dR_t}{dI_t} = a - bI_t$$

# Bonds

- Net bonds have a rate of return equal to $i$, so that earnings from net bonds are given by the equation

$$Y_t = i\tilde{B}_t$$

- Finally, the stock of net securities is equal to the portion ($\rho$) of net bonds of the previous period that has not expired (or has been renewed) plus new net bonds:

$$\tilde{B}_t = \rho\tilde{B}_{t-1} + \Delta\tilde{B}_t$$

# Bonds

- Net bonds have a rate of return equal to $i$, so that earnings from net bonds are given by the equation

$$Y_t = i\tilde{B}_t$$

- Finally, the stock of net securities is equal to the portion ($\rho$) of net bonds of the previous period that has not expired (or has been renewed) plus new net bonds:

$$\tilde{B}_t = \rho\tilde{B}_{t-1} + \Delta\tilde{B}_t$$

# Equilibria

- We analyze the two-dimensional map $(\Pi_t, \widetilde{B}_t) = T(\Pi_{t-1}, \widetilde{B}_{t-1})$

$$T : \begin{cases} \Pi_t = -\frac{b}{2}\alpha_2^2\Pi_{t-1}^2 + (1 + a\alpha_2 + i\alpha_3)\Pi_{t-1} + (i\rho + 1 - \rho)\widetilde{B}_{t-1} \\ \widetilde{B}_t = \rho\widetilde{B}_{t-1} + \alpha_3\Pi_{t-1} \end{cases}$$

  where $\alpha_3 = 1 - \alpha_1 - \alpha_2$.
- To simplify the notation let us use $B$ in place of $\widetilde{B}$

# Equilibria

- The equilibria are given by the origin $(0,0)$, and $(\Pi^*, B^*)$ where

$$B^* = \frac{\alpha_3}{1-\rho}\Pi^*$$

and $\Pi^*$

$$\Pi^* = \frac{2}{b\alpha_2^2}\left[a\alpha_2 + i\alpha_3 + (i\rho + 1 - \rho)\frac{\alpha_3}{1-\rho}\right]$$

- so $(\Pi^*, B^*) = (\Pi^*, \frac{\alpha_3}{1-\rho}\Pi^*)$ is the equilibrium we are interested in.

# Jacobian Matrix

- The Jacobian Matrix is given by

$$J(\Pi^*, B^*) = \begin{bmatrix} -2[a\alpha_2 + i\alpha_3 + (i\rho + 1 - \rho)\frac{\alpha_3}{1-\rho}] + (1 + a\alpha_2 + i\alpha_3) & (i\rho + 1 - \rho) \\ \alpha_3 & \rho \end{bmatrix}$$
$$= \begin{bmatrix} 1 - (a\alpha_2 + i\alpha_3) - \frac{2\alpha_3}{1-\rho}(i\rho + 1 - \rho) & (i\rho + 1 - \rho) \\ \alpha_3 & \rho \end{bmatrix}$$

- The Jacobian Matrix evaluated at the equilibrium is independent on the parameter $b$, that is, adjustment costs do not affect the stability of final equilibria.

# Stability conditions

- The equilibrium is **stable** when:
  1. $detJ^* < 1$,
  2. $\mathcal{P}(1) > 0$,
  3. $\mathcal{P}(-1) > 0$.

1 The condition $detJ^* < 1$ leads to

$$-(1-\rho) - \rho i \alpha_3 - \alpha_3(i\rho + 1 - \rho)\frac{1+\rho}{1-\rho} < \rho a \alpha_2$$

which is always satisfied (thus we cannot have a Neimark Sacker bifurcation).

# Stability conditions

- The equilibrium is **stable** when:
  1. $detJ^* < 1$,
  2. $\mathcal{P}(1) > 0$,
  3. $\mathcal{P}(-1) > 0$.

1 The condition $detJ^* < 1$ leads to

$$-(1-\rho) - \rho i \alpha_3 - \alpha_3(i\rho + 1 - \rho)\frac{1+\rho}{1-\rho} < \rho a \alpha_2$$

which is always satisfied (thus we cannot have a Neimark Sacker bifurcation).

# Stability conditions

- The equilibrium is **stable** when:
  1. $detJ^* < 1$,
  2. $\mathcal{P}(1) > 0$,
  3. $\mathcal{P}(-1) > 0$.

1 The condition $detJ^* < 1$ leads to

$$-(1-\rho) - \rho i \alpha_3 - \alpha_3 (i\rho + 1 - \rho)\frac{1+\rho}{1-\rho} < \rho a \alpha_2$$

which is always satisfied (thus we cannot have a Neimark Sacker bifurcation).

# Stability conditions

- The equilibrium is **stable** when:
  1. $detJ^* < 1$,
  2. $\mathcal{P}(1) > 0$,
  3. $\mathcal{P}(-1) > 0$.

1 The condition $detJ^* < 1$ leads to

$$-(1-\rho) - \rho i \alpha_3 - \alpha_3(i\rho + 1 - \rho)\frac{1+\rho}{1-\rho} < \rho a \alpha_2$$

which is always satisfied (thus we cannot have a Neimark Sacker bifurcation).

2 The condition $\mathcal{P}(1) > 0$ is always satisfied as

$$\mathcal{P}(1) = 1 - TrJ^* + DetJ^* = (1-\rho)(a\alpha_2 + i\alpha_3) + \alpha_3(i\rho + 1 - \rho)$$

3 The condition $\mathcal{P}(-1) > 0$ may be satisfied or not as

$$\mathcal{P}(-1) = 1 + TrJ^* + DetJ^* = 2(1+\rho) - (1+\rho)(a\alpha_2 + i\alpha_3) - \alpha_3(i\rho + 1 - \rho)\frac{3+\rho}{1-\rho}$$

- Thus, the condition $\mathcal{P}(-1) = 0$ leads to a flip bifurcation of the fixed point.

## Stability

2 The condition $\mathcal{P}(1) > 0$ is always satisfied as

$$\begin{aligned}\mathcal{P}(1) &= 1 - TrJ^* + DetJ^* \\ &= (1-\rho)(a\alpha_2 + i\alpha_3) + \alpha_3(i\rho + 1 - \rho)\end{aligned}$$

3 The condition $\mathcal{P}(-1) > 0$ may be satisfied or not as

$$\begin{aligned}\mathcal{P}(-1) &= 1 + TrJ^* + DetJ^* \\ &= 2(1+\rho) - (1+\rho)(a\alpha_2 + i\alpha_3) - \alpha_3(i\rho + 1 - \rho)\frac{3+\rho}{1-\rho}\end{aligned}$$

- Thus, the condition $\mathcal{P}(-1) = 0$ leads to a flip bifurcation of the fixed point.

# Bifurcation condition

- The bifurcation condition $\mathcal{P}(-1) = 0$ can be written as

$$\alpha_2 = -\alpha_1 \frac{(1-\rho^2)i + (i\rho + 1 - \rho)(3+\rho)}{(1-\rho^2)(i-a) + (i\rho + 1 - \rho)(3+\rho)} + \frac{(1-\rho^2)(i-2) + (i\rho + 1 - \rho)(3+\rho)}{(1-\rho^2)(i-a) + (i\rho + 1 - \rho)(3+\rho)}$$

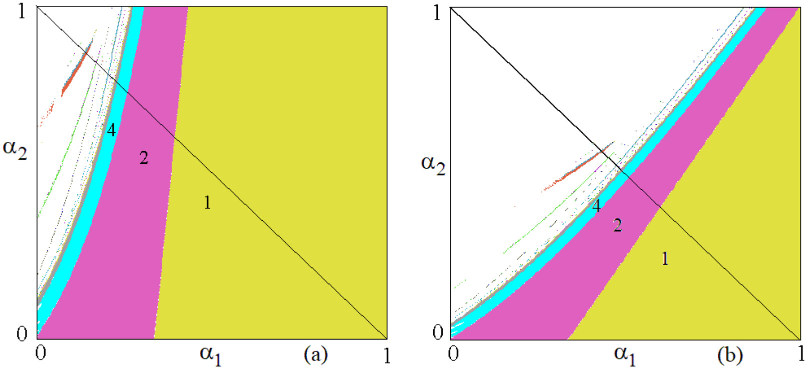
- This line divides the stability region of the fix point from period-2 cycle. The slope depends on $a$, the return of investment.

# Bifurcation diagrams

- At fixed parameters $b = 0.05$, $i = 0.02$, $\rho = 0.96$, the bifurcation diagram in the $(\alpha_1, \alpha_2)$ parameter plane is shown in the following Fig1(a) for $a = 3.3$, and in Fig1(b) for $a = 5$.
- On the diagonal it is $\alpha_1 + \alpha_2 = 1$ so that above it we have $\alpha_3 < 0$, that is firm is either borrowing from the markets or issuing bonds.
- It can be seen that as long as the parameter $a$ increases, the stability region of the fixed point is reduced, and a wider region with chaos occurs.
- As the parameter $a$ captures the return of investment, larger $a$ seems to determine investment in riskier-asset and potentially over investment.
- Said differently, for larger $a$, meaning that investment earns at larger marginal returns, the system becomes more unstable or chaotic.
- As long as firm holds too much cash, and the larger $a$, the firm invests in riskier assets, generating instability.

# Cash, Risky investment and chaos

Figure: **1. The impact of parameter** $a$

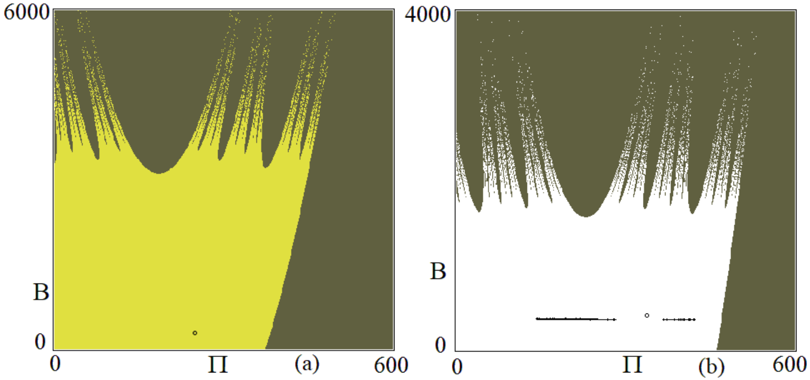


# Bifurcation diagrams

- In order to see the phase space for $\alpha_3 > 0$ we show two cases in the following Figure 2, at $b = 0.05$, $i = 0.02$, $\rho = 0.96$, $\alpha_1 = 0.4$, $\alpha_2 = 0.55$.
- In (a), $a = 3.3$ the fixed point $(\Pi^*, B^*) = (249.92, 312.4)$ is attracting and in yellow we show its basin of attraction, showing that it is quite robust.
- In (b), $a = 4.5$, the fixed point $(\Pi^*, B^*) = (337.19, 421, 87)$ is unstable and we have a chaotic attracting set whose basin is in white.
- The gray region denotes points having a divergent trajectory.
- In this case, it can be seen that in the chaotic region although there is a wire chaotic range in the values of Π, the values of B changes inside a small strip.

# Risky investment and chaos

Figure: **2. The phase space in the case of positive net bonds** ($\alpha_3 > 0$)

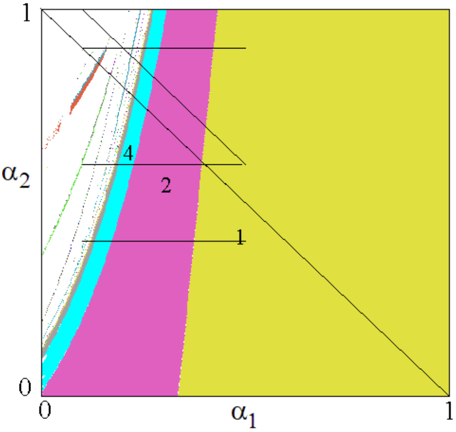


# Bifurcation diagrams

- This can be better appreciated looking at the one-dimensional bifurcation diagrams as a function of only one parameter.
- In the Figures (4)-(7) below we fix $\alpha_2$ and vary $\alpha_1$ in the interval $(.1, .5)$ along the lines evidenced in the following Figure 3, at $a = 3.3$.
- In the two-dimensional bifurcation diagram at fixed $b = 0.05$, $i = 0.02$, $\rho = 0.96$, $a = 3.3$ we show the horizhontal segments at $\alpha_2 = 0.9$, $\alpha_2 = 0.6$, $\alpha_2 = 0.4$, and the segment on $\alpha_2 = -\alpha_1 + 1.1$ along which the one-dimensional bifurcation diagram as a function of $\alpha_1$ is shown.

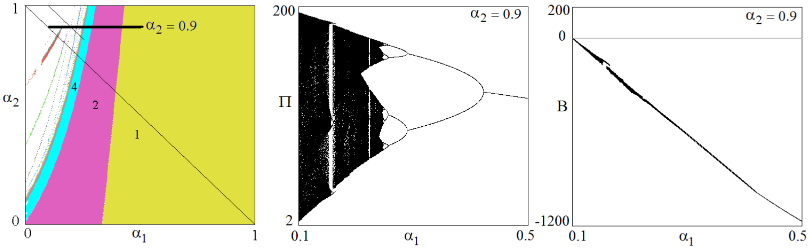
# Dividend policy, firm indebtedness and chaos

Figure: **3. Alternative cash allocations**



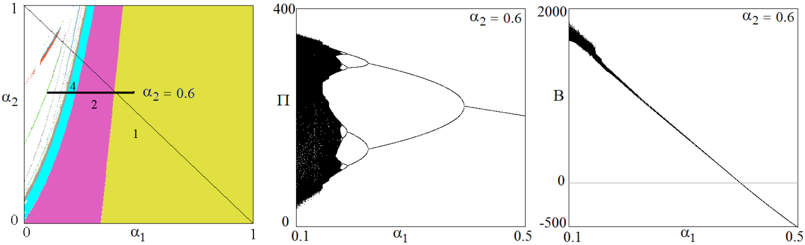
# Dividend policy, firm indebtedness and chaos

Figure: **4. High investment, dividend policy and transition to chaos**



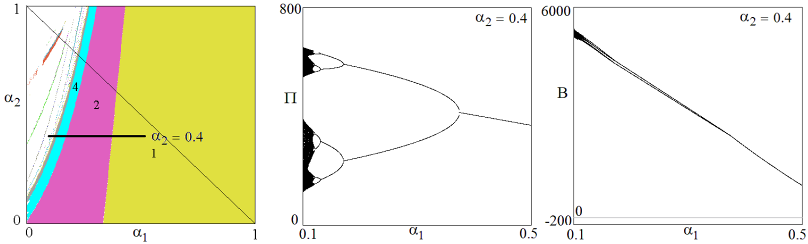
# Dividend policy, firm indebtedness and chaos

Figure: **5. Investment, dividend policy and transition to chaos**



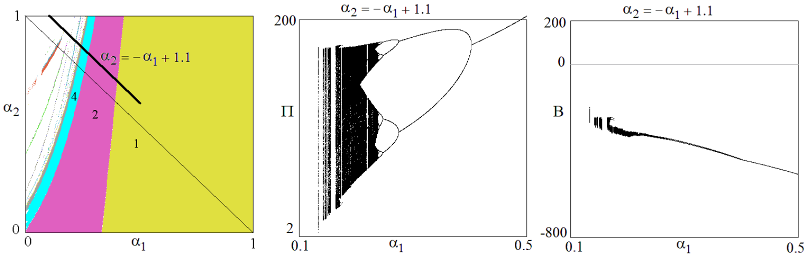
# Dividend policy, net bonds and chaos

Figure: **6. Net bonds, dividend policy and transition to chaos**



# Dividend and investment policy, firm indebtedness and chaos

Figure: **7. Investment and dividend policy and transition to chaos**

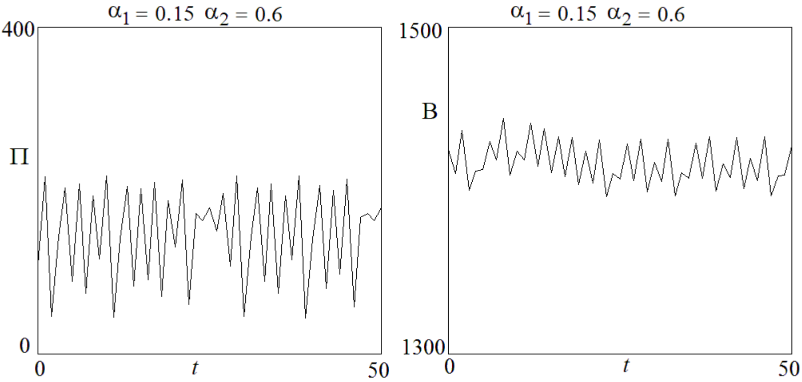


# Versus-time trajectory

- The versus-time trajectory is shown in two cases in the following Figures (8) and (9), at fixed parameters $a = 3.3$, $b = 0.05$, $i = 0.02$, $\rho = 0.96$
- We obtain that the chaotic regime is never periodic
- Furthermore, cash ($\Pi$) fluctuates more than net bonds ($B$), the variation of which is always in a small range.

# Versus-time trajectory

Figure: **8. Investment, dividend policy and versus-time trajectories for Cash and net Bonds**



# Versus-time trajectory

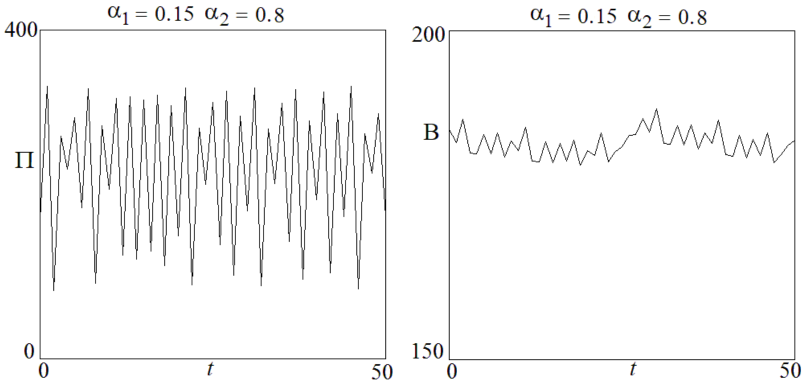



Figure: **9. High investment, dividend policy and versus-time trajectories for Cash and net Bonds**

# Conclusion

- This paper was motivated by the attempt to answer the question whether or not too much finance might generate instability
- We started by modeling firm cash holding, and firm decisions related to dividend, financial policies and real investment decisions
- We obtain equilibria characterized by chaotic time paths, showing that too much finance, by providing wrong signals to the markets, might generate instability
- Next step: testing theoretical predictions with firm data.

# Conclusion

- This paper was motivated by the attempt to answer the question whether or not too much finance might generate instability
- We started by modeling firm cash holding, and firm decisions related to dividend, financial policies and real investment decisions
- We obtain equilibria characterized by chaotic time paths, showing that too much finance, by providing wrong signals to the markets, might generate instability
- Next step: testing theoretical predictions with firm data.

# Conclusion

- This paper was motivated by the attempt to answer the question whether or not too much finance might generate instability
- We started by modeling firm cash holding, and firm decisions related to dividend, financial policies and real investment decisions
- We obtain equilibria characterized by chaotic time paths, showing that too much finance, by providing wrong signals to the markets, might generate instability
- Next step: testing theoretical predictions with firm data.

# Conclusion

- This paper was motivated by the attempt to answer the question whether or not too much finance might generate instability
- We started by modeling firm cash holding, and firm decisions related to dividend, financial policies and real investment decisions
- We obtain equilibria characterized by chaotic time paths, showing that too much finance, by providing wrong signals to the markets, might generate instability
- Next step: testing theoretical predictions with firm data.

# Thanks!!

**Thanks for your attention.**

# Cash holdings in Italian medium- and large-sized firms

◂ Go Back

Figure: **Firms' investment and liquidity in Italy. Index numbers (2009=100). MedioBanca (2019)**