Stock market participation and endogenous boom-bust dynamics: momentum, mispricing and risk

Roberto Dieci*, Noemi Schmitt** and Frank Westerhoff**

* University of Bologna ** University of Bamberg

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Outline

1. Brief introduction and sketch of the main findings.
2. Model setup / building blocks:
   - tradeoffs between trend, 'value' and risk;
   - investors' market entry and exit decisions and impact on prices;
   - updating of risk beliefs.
3. Dynamical system: Fundamental Steady State (FSS) and Risk-Ajusted Discount Rate (RADR), stability properties, determinants of endogenous asset price fluctuations.
4. Numerical investigations: participation waves, boom-bust dynamics, coexisting attractors and permanent shifts of price levels.
5. Conclusion

1. Introduction, motivation, main findings

- A fairly general market entry model that can foster our understanding of the boom-bust behavior of stock markets, whilst remaining *analytically tractable* and consistent with *standard valuation* approaches and risk-return relationships at the steady state.
- Investors' entry / exit decisions depend on three core principles: (*i*) *price trend / momentum*; (*ii*) *fundamental conditions* and (*iii*) *risk*. However, investors are also subject to *herding behavior*.
- The model's FSS reflects standard present-value relations between average dividends and the risk-adjusted interest rate.
- Substantial boom-bust dynamics are set in motion if sufficiently many outside investors react strongly to the market's momentum.
- Investors' reaction to endogenous stock market risk leads to non trivial and harmful effects, such as sudden and permanent shifts of the level around which the market oscillates.

2. Model setup - notation

- A speculative / risky asset (e.g. stock market) vs. a risk-free alternative investment (similar to SW 2017, DSW 2018);
- n_t: number of investors active in the speculative asset market in period t;
- N: total number of investors (total *market participation*);
- $N - n_t$: number of outside investors (opting for the safe alternative);
- $p_t = h(n_t)$, $h' > 0$: asset price in t; d: (average) dividend on the risky asset;
- A^B: (constant) *attractiveness* of the safe investment alternative (an increasing function of the risk-free rate r);
- A_t: *attractiveness* of the asset market in period t (broadly related to the asset's return potential and risk).

2. Model setup - market attractiveness

- The attractiveness of the risky asset market in period t:

$$A_t = \Phi(\rho_t, \delta_t, v_t), \qquad \Phi_\rho, \Phi_\delta > 0, \quad \Phi_v < 0.$$

  - $\rho_t := (p_t - p_{t-1})/p_{t-1}$: *price return* (indicates the *trend*, can be generalized to longer time intervals);
  - $\delta_t := d/p_t$: *dividend/price (D/P)* ratio (fundamental conditions / misalignments)
  - v_t: (*perceived*) price variance in period t ($s_t := \sqrt{v_t}$: s.d.).
- Both *trend* (ρ_t) and *D/P ratio* (δ_t) are positively related to the asset's return potential (*expected return*), in investors' view.
- Perceived variance (v_t) is related to *risk*;
- A *tradeoff* between *trend* and *fundamental conditions* and between *expected return* and *risk* (in the spirit of mean-variance utility).

2. Model setup - replicator dynamics

- *Exponential replicator dynamics* of investors participating in the risky and safe investment alternatives:

$$n_{t+1} = N \frac{n_t \exp(\lambda A_t)}{n_t \exp(\lambda A_t) + (N - n_t) \exp(\lambda A^B)},$$

  or equivalently (in terms of proportions):

$$x_{t+1} = \frac{x_t \exp(\lambda A_t)}{x_t \exp(\lambda A_t) + (1 - x_t) \exp(\lambda A^B)}, \qquad x_t := \frac{n_t}{N}$$

- Switching is mainly governed by *relative attractiveness* (but *herding behavior* also plays a role);
- Trend ρ_t and D/P ratio δ_t depend on n_t, n_{t-1}, v_t.

2. Model setup - belief updating rules

- Variance beliefs determined through time averages evolving via adaptive rules (see, e.g. the work surveyed by Dieci and He (2018)):
- $v_t = v_F + v_{p,t}$, where v_F is a baseline (exogenous) level related to 'fundamental risk';
- $v_{p,t}$ is *updated recursively* according to:

$$v_{p,t} = m v_{p,t-1} + m(1-m)(p_t - u_{t-1})^2 \qquad (0 < m < 1)$$

where: $u_t = m u_{t-1} + (1-m) p_t$. This is equivalent to:

$$u_t = \sum_{s=0}^{\infty} \omega_s p_{t-s} \ , \qquad v_{p,t} = \sum_{s=0}^{\infty} \omega_s (p_{t-s} - u_t)^2 \ ,$$

$$\omega_s := (1-m) m^s \ , \qquad \sum_{s=0}^{\infty} \omega_s = 1$$

3. Dynamical system

- Formally, a 4-dimensional nonlinear dynamical system in n_t, $z_t := n_{t-1}$, u_t, $v_{p,t}$:

$$\begin{aligned} n_{t+1} &= F(n_t, z_t, v_{p,t}) = N \frac{n_t \exp(\lambda A_t)}{n_t \exp(\lambda A_t) + (N - n_t) \exp(\lambda A^B)}, \\ z_{t+1} &= n_t, \\ u_{t+1} &= m u_t + (1-m) h(F(n_t, z_t, v_{p,t})) \\ v_{p,t+1} &= m v_{p,t} + m(1-m) \left[h(F(n_t, z_t, v_{p,t}) - u_t) \right]^2 \end{aligned}$$

  where:

$$A_t = \Phi(\rho_t, \delta_t, v_t), \quad \rho_t = \frac{h(n_t)}{h(z_t)} - 1, \quad \delta_t = \frac{d}{h(n_t)}, \quad v_t = v_F + v_{p,t}$$

- Main endogenous forces:
  - relative performances (attractiveness) and herding
  - coexisting *positive and negative feedback* mechanisms
  - endogenous (adaptive) updating rules

3. Dynamical system - Fundamental Steady State - FSS

- *Analytical* characterization / stability properties of the *Fundamental Steady State - FSS* in a very general setting.
- Steady state dynamics requires: $\bar{z} = \bar{n}$, $\bar{u} = \bar{p} = h(\bar{n})$, $\bar{\rho} = 0$, $\bar{v}_p = 0$ (and therefore $\bar{v} = v_F$).
- A unique interior steady state (*FSS*) characterized by *equal attractiveness* of the risky and safe assets, $\bar{A} = A^B$, namely:

$$\bar{A} = \Phi(0, \bar{\delta}, v_F) := \varphi(\bar{\delta}) = A^B$$

- As a consequence, the return of the risky asset at the FSS (= D/P ratio) is given by:

$$\bar{\delta} := \frac{d}{\bar{p}} = \frac{d}{h(\bar{n})} = \varphi^{-1}(A^B) := r^a.$$

3. Dynamical system - Fundamental Price and RADR

- r^a is a *risk-adjusted* discount rate (*RADR*), strictly *increasing* with A^B, v_F and the risk-related parameters associated with Φ_v.
- The *fundamental price* (*FP*) $\bar{p}$ obeys the standard dividend discount formula, $\bar{n}$ is determined accordingly:

$$\bar{p} = \frac{d}{r^a}, \qquad \bar{n} = h^{-1}(\bar{p}) = h^{-1}\left(\frac{d}{r^a}\right).$$

- $\bar{p}$ and r^a thus emerge endogenously from the steady-state '*no-arbitrage* condition' $\bar{A} = A^B$, by taking *adjustment for risk* into account.
- Both $\bar{p}$ and $\bar{n}$ decrease with r^a.

3. Dynamical system - FSS stability

- A *general* stability result in terms of the partials of the attractiveness, Φ_ρ, Φ_δ, Φ_v (at the FSS).
- Define $\beta := \Phi_\rho > 0$, $\gamma := \Phi_\delta > 0$, $\theta := -\ \Phi_v > 0$.
- Define also $\bar{x} := \bar{n}/N$, $\varepsilon := \bar{n}h'(\bar{n})/h(\bar{n}) = \bar{n}h'(\bar{n})/\bar{p}$ (*elasticity* of p with respect to participation n, at the FSS)
- FSS is Locally Asymptotically Stable (*LAS*) iff:

$$\frac{\gamma r^a}{2} - \frac{1}{\lambda\varepsilon(1-\bar{x})} < \beta < \frac{1}{\lambda\varepsilon(1-\bar{x})}.$$

- Violation of the right (resp. left) inequality results in a *Neimark-Sacker* bifurcation (resp. *Flip* bifurcation).

3. Dynamical system - a closer look at the NS bifurcation

- The *Neimark-Sacker bifurcation boundary*:

$$\beta\lambda\varepsilon(1-\bar{x}) < 1$$

- FSS may lose stability via a NS bifurcation and give rise to periodic or quasi-periodic fluctuations for *sufficiently large*
  - reaction to recent trends (β) and switching intensity (λ)
  - asset price response to changes of participation in the asset market (ε)
  - proportion of outside investors (potential new entrants):

$$\bar{y} := 1 - \bar{x} = 1 - \frac{\bar{n}}{N} = 1 - \frac{1}{N} h^{-1}\left(\frac{d}{r^a}\right)$$

where $\bar{y}$ depends positively on total market participation (N) and the risk-adjusted required return (r^a) and where r^a, on its turn, depends positively on A^B, v_F and the other risk-sensitivity parameters.

4.Numerical investigation - baseline parameters

- Baseline parameter setting for an *illustrative* example: $d = 1.2$, $N = 200$, $\lambda = 1$;
- Memory parameter: $m = 0.95$.
- Attractiveness of the asset market:

$$A_t = \Phi(\rho_t, \delta_t, v_t) = \mu \arctan\left(\frac{\beta}{\mu}\rho_t\right) + \gamma\delta_t - \psi\sqrt{v_t}$$

  where: $\mu := \frac{2\kappa}{\pi}$, $\beta = \Phi_\rho$, $\gamma = \Phi_\delta$, κ, $\psi > 0$. We set: $\gamma = 20$, $\kappa = 0.05$;
- Price determined as: $p_t = h(n_t) = an^q$. Baseline values are: $a = 1$, $q = \varepsilon = 1$;

4.Numerical investigation - baseline parameters (ctd.)

- Risk-free return: $r = 0.01$;
- Attractiveness of the safe investment: $A^B = \Phi(0, r, 0) = \gamma r = 0.01\gamma$.
- Risk and risk-related parameters (note that $r^a = r + \psi\sqrt{v_F}/\gamma$):
  - $s_F = \sqrt{v_F} = \sigma p_F$ ($p_F := d/r$);
  - $\psi = \nu\gamma/p_F$ ($\nu > 0$)
  - $\Longrightarrow r^a = r + \nu\sigma$, where we set $\sigma = 0.02$, $\nu = 0.1$ and therefore $r^a = 0.012$
- As a consequence, $\bar{n} = \bar{p} = 100$, $\bar{y} := 1 - \bar{x} = 0.5$, the NS bifurcation value for β is $\beta^{NS} = 2$.

4. Numerical investigation - dynamics beyond the NS boundary

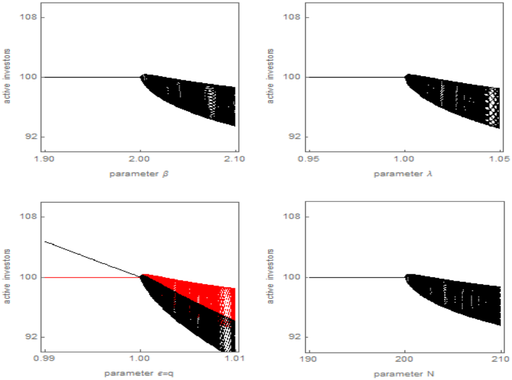

Figure: *Neimark Sacker bifurcation*. Bifurcation diagrams of n_t ($= p_t$, except in the bottom-left panel, where p is in red) against reaction to trend (β, top left), switching parameter (λ, top right), elasticity of the price ($\varepsilon = q$, bottom left), total n. of investors (N, bottom right). Parameters as in our baseline selection.

4. Numerical investigation - dynamics beyond the NS boundary

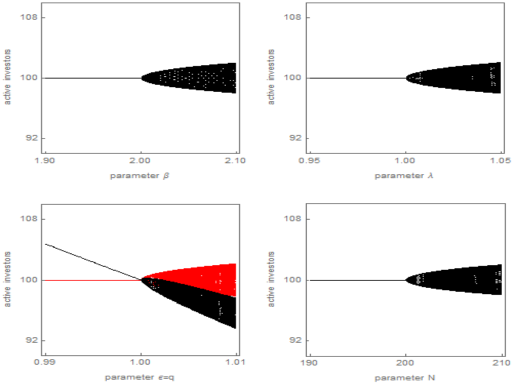

Figure: *Neimark Sacker bifurcation without endogenous risk perceptions* ($v_{p,0} = 0$, $m = 0$). Bifurcation diagrams of n_t ($= p_t$, except in the bottom-left panel, where p in red) against parameter β (top left), λ (top right), $\varepsilon = q$ (bottom left), N (bottom right). Other parameters as in our baseline selection.

4. Numerical investigation - NS bifurcation and coexisting attractors

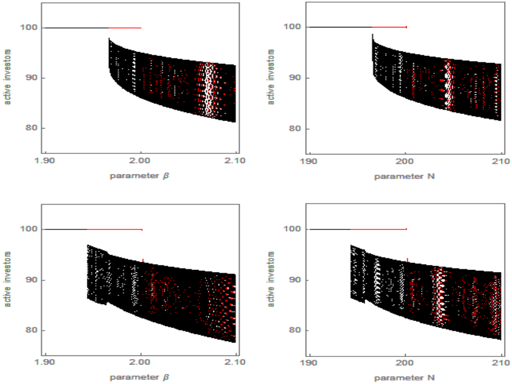

Figure: *Neimark Sacker bifurcation and coexisting attractors*. Bifurcation diagrams of $n_t = p_t$ against sensitivity to trend (left panels) and total market participation (right panels). Parameters as in our baseline selection, except $\kappa = 0.09$ (top panels) and $\gamma = 5$ (bottom panel).

4. Numerical investigation - NS bifurcation and coexisting attractors

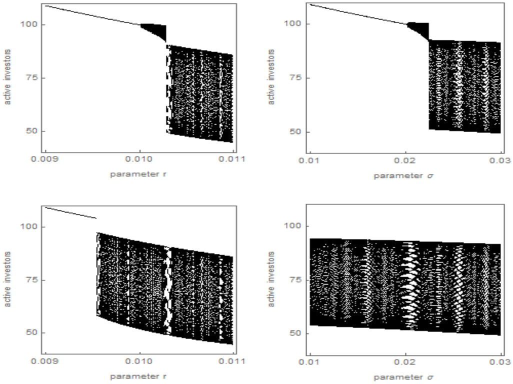

Figure: *Neimark Sacker bifurcation and risk-adjusted discount factor.* Effects of increasing discount factors through parameters r (left) and σ (right). Base parameter setting, except $\kappa = 0.45$ and $m = 0.5$. Top and bottom panels differ with respect to initial conditions.

4. Numerical investigation - Functioning of the model

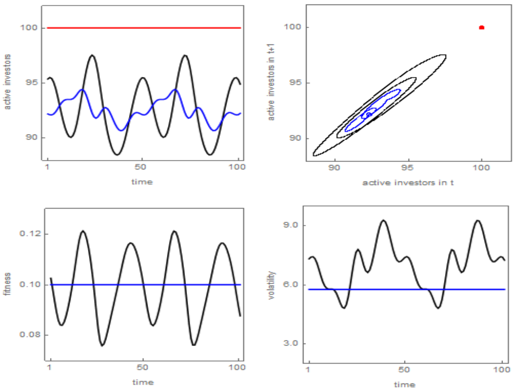

Figure: *Functioning of model beyond the NS boundary*. Top-left: time paths of n_t (p_t) Top-right: projection in the plane (n_t, n_{t+1}). Bottom left: time path of A_t. Bottom-right: time path of v_t. Constant FSS levels are in red (top), or in blue (bottom). The blue paths / orbits in the top panels correspond to a 'time-varying' FP (dividends discounted at the *current* RADR). Base parameter setting except that $\beta = 2.05$ and $\gamma = 10$.

4. Conclusion

- A general stock market entry model; investors look at *price trends*, *quantitative anchors*, *risk*; market entry / exit is governed by the markets' relative attractiveness and herding behavior, stock price reacts positively to market participation.
- Steady-state dynamics is consistent with standard asset valuation and adjustment for risk.
- Endogenous boom-bust dynamics via a *supercritical* or *subcritical* Neimark-Sacker (NS) bifurcation arise - amongst others - if sufficiently many outside investors react strongly to market momentum.
- Due to investors' reaction to endogenous stock market risk, NS bifurcation leads to an increase of volatility and a simultaneous downward shift of the price level, often in a *sharp* manner.
- Stock market participation waves in the presence of risk can create substantial and harmful boom-bust dynamics.