An overlapping generations model with two types of agents and shift in behaviour

Pasquale Commendatore (University of Naples “Federico II”, Italy)

Ingrid Kubin (Vienna University of Economics and BA, Austria)

Iryna Sushko (NASU and Kyiv School of Economics, Ukraine)

NED2019
Kyiv
Ukraine
Sept 4-6

Contents

Introduction

- We present an overlapping generations model describing an economy in which two types of agents may co-exist: ‘workers’ and capitalists’ (Pasinetti, 1962).
- Workers and capitalists save on the basis of rational choices (Foley & Michl, 1999; Michl, 2009; Commendatore & Palmisani, 2009).
- Workers face a finite time horizon and base their consumption choices on a life-cycle motive.
- Capitalists behave like an infinitely-lived dynasty.

- Capitalists only source of income is profit.
- Workers main source of income is wages. However, depending on their income workers may have a switch in behaviour:
- above a certain threshold they decide to leave bequests to the offspring according to a ‘warm glow’ motive (see Andreoni, 1989, 1990).
- Empirical literature confirms that households with higher levels of (lifetime) income typically leave very large bequests (De Nardi, 2004).

- The resulting model is in three dimensions with a discontinuity.
- We consider a special case in which a distinct class of capitalists does no exist. The resulting model is 2-dimensional with a discontinuity.
- Notice that when workers' income is sufficiently high, so that they never change their behaviour altruism never sets in, the 2-D model reduces to that proposed by Michel and de La Croix (2000) and Chen et al. (2008) with myopic expectations.

- We will study the dynamics properties of the 2-D model with switch in behaviour both in relation to the local stability properties and to the global dynamics verifying how the threshold impinges on those properties.
- We will also verify how changes in workers behaviour affects capital accumulation. This will be crucial, when capitalists are reintroduced, to study the overall effects on the distribution of capital between the two groups.

Full model (sketch)

Capitalists

- Are represented as a single dynasty with and infinite time horizon (Barro, 1974).
- The only source of income is profit.
- Given initial wealth $K_{c,0}$, at the beginning of period 0 they choose consumption quantities $\left(C_{c,0}, C_{c,1}, C_{c,2}, \dots, C_{c,t,} \dots\right) = (C_{c,t})_0^\infty$ in order to solve:

$$\max \sum_{t=0}^{\infty} \beta_c^t \mathrm{U}(C_{c,t})$$

$$\text{subject to } C_{c,t} + K_{c,t+1} \leq (1 + r_t - \delta) K_{c,t}$$

- where $0 \leq \delta \leq 1$ is the depreciation rate and $0 < \beta_c < 1$ is capitalists time discount factor.
- $K_{c,t}$ is the capital of capitalist at point of time t.
- For future reference $k_{c,t}$ is the capitalists' capital per worker.
- N.B. Capitalists have perfect foresight.

Workers

- The population of workers has an overlapping generations structure
- Each generation faces a finite time horizon composed of two periods.
- Each individual is active, working and earning a wage, when "young" (i.e. in the first period of life).
- And she is in retirement when "old" (i.e. in the second period of life).
- In each period *t* a young worker inelastically supplies one unit of labour at the wage rate w_t.
- L_t is the overall number of workers, *n* is the rate of growth, where $n > -1$. Thus $L_{t+1} = (1 + n)L_t$.
- N.B. Workers have myopic foresight.

Workers

- If workers' income is low, they save out of income only to be able to consume during retirement according to a life cycle motive.
- There is not intergenerational transfer of wealth (Diamond, 1965).
- A young worker chooses the quantities of current $c_{w,t}$ and future consumption $c_{w,t+1}$ solving the following constrained utility maximization problem:

$$\max[U(c_{w,t}) + \beta_{w1} U(c_{w,t+1})]$$

$$\text{subject to } \; c_{w,t} + \frac{c_{w,t+1}}{1+r_t-\delta} \leq w_t + b_t,$$

- where $0 \leq \beta_{w1} < 1$ is the workers' consumption discount factor.

Workers

- If workers' income is high, workers decide to behave altruistically towards their offspring
- Parents value the bequest per se following the "warm glow" (joy of giving) approach (Andreoni 1989, 1990)
- The amount of the bequest is one of the arguments of the workers' intertemporal utility function. A single worker solves:

$$\max U(c_{w,t}) + \beta_{w1} U(c_{w,t+1}) + \beta_{w2} U((1+n) b_{t+1})$$

- Subject to:
- $c_{w,t} + \frac{c_{w,t+1}}{1+r_t-\delta} + \frac{b_{t+1}(1+n)}{1+r_t-\delta} \leq w_t + b_t,$
- where $0 \leq \beta_{w2} < 1$ is the discount factor workers apply to bequests, with $\beta_{w2} \leq \beta_{w1}$.

Reduced version
(no capitalists)

- In the full version, the model is three-dimensional (the state variables are: capitalists' capital, workers' capital and workers' bequests) with a discontinuity.
- We simplify by assuming $k_{c,t} = 0$ which holds for all t (no capitalists), the resulting model is two-dimensional (the state variables are reduced to capital and bequests).
- We drop the subscript w.

- We assume a CIES Utility function (σ intertemporal elasticity of substitution):

$$U(c) = f(x) = \begin{cases} \left(1 - \frac{1}{\sigma}\right)^{-1} c^{1-\frac{1}{\sigma}} & for \quad \sigma > 0 \; and \; \sigma \neq 1 \\ \ln(c) & for \quad \sigma = 1 \end{cases}$$

- and a Cobb-Douglas production function:

$$f(k) = \alpha A k^{\alpha}$$

with $0 < \alpha < 1$ and $A > 0$

- If $w_t + b_t < \bar{y}$ (i.e. agent's income is low, with $\bar{y} \geq 0$) solutions satisfy the condition:
- $c_t = \frac{\rho_t(w_t+b_t)}{\rho_t+(\rho_t\beta_1)^\sigma}$
- $c_{t+1} = \frac{\rho_t(\rho_t\beta_1)^\sigma(w_t+b_t)}{\rho_t+(\rho_t\beta_1)^\sigma}$
- A single agent saving corresponds to:
- $s_t = w_t + b_t - c_t = \frac{(\rho_t\beta_1)^\sigma(w_t+b_t)}{\rho_t+(\rho_t\beta_1)^\sigma}$

where $\rho_t = 1 + r_t - \delta$

- If $w_t + b_t \geq \bar{y}$ (i.e. agent's income is high, with $\bar{y} \geq 0$) solutions satisfy the condition:
- $c_t = \frac{\rho_t(w_t+b_t)}{\rho_t+(\rho_t\beta_1)^\sigma+(\rho_t\beta_2)^\sigma}$
- $c_{t+1} = \frac{\rho_t(\rho_t\beta_1)^\sigma(w_t+b_t)}{\rho_t+(\rho_t\beta_1)^\sigma+(\rho_t\beta_2)^\sigma}$
- $b_{t+1}(1+n) = \frac{\rho_t(\rho_t\beta_2)^\sigma(w_t+b_t)}{\rho_t+(\rho_t\beta_1)^\sigma+(\rho_t\beta_2)^\sigma}$
- A single agent saving corresponds to:
- $s_t = w_t + b_t - c_t = \frac{[(\rho_t\beta_1)^\sigma+(\rho_t\beta_2)^\sigma](w_t+b_t)}{\rho_t+(\rho_t\beta_1)^\sigma+(\rho_t\beta_2)^\sigma}$

where $\rho_t = 1 + r_t - \delta$

Equlibrium conditions in the labour and capital markets

Given the production function

$$f(k_t) = Ak_t^{\alpha}$$

- Equilibrium in the capital market involves:

$$r_t = f'(k_t) = \alpha A k_t^{\alpha-1}$$

- Equilibrium in the labour market involves:

$$w_t = f(k_t) - f'(k_t)k_t = (1-\alpha)Ak_t^{\alpha}$$

Dynamic equations

- Considering that $L_{t+1} = (1+n)L_t$ and using the equilibrium condition savings = investments,
- for $w_t + b_t < \bar{y}$ we can write:

$$k_{t+1} = \frac{1}{1+n} s_t = \frac{1}{1+n} \frac{(\rho_t \beta_1)^\sigma (w_t + b_t)}{\rho_t + (\rho_t \beta_1)^\sigma}$$

$$b_{t+1} = 0$$

- for $w_t + b_t \geq \bar{y}$ we can write:

$$k_{t+1} = \frac{1}{1+n} s_t = \frac{1}{1+n} \frac{[(\rho_t \beta_1)^\sigma + (\rho_t \beta_2)^\sigma](w_t + b_t)}{\rho_t + (\rho_t \beta_1)^\sigma + (\rho_t \beta_2)^\sigma}$$

$$b_{t+1} = \frac{1}{1+n} \frac{\rho_t (\rho_t \beta_2)^\sigma (w_t + b_t)}{\rho_t + (\rho_t \beta_1)^\sigma + (\rho_t \beta_2)^\sigma}$$

Dropping the time subscripts and letting $x = k$ and $y = b$ the dynamic system is represented by the following map *F*

The map F is defined as follows: [illegible] 1 [illegible] Ax [illegible], w [illegible] Ax [illegible] and

F_1 : $\begin{pmatrix} x \\ y \end{pmatrix}$ [illegible] $\begin{pmatrix} \text{[illegible]} \\ 0 \end{pmatrix}$ if y [illegible] $\bar{y}$ [illegible] w

F_2 : $\begin{pmatrix} x \\ y \end{pmatrix}$ [illegible] $\begin{pmatrix} \text{[illegible]} \\ \text{[illegible]} \end{pmatrix}$ if y [illegible] $\bar{y}$ [illegible] w

Parameters

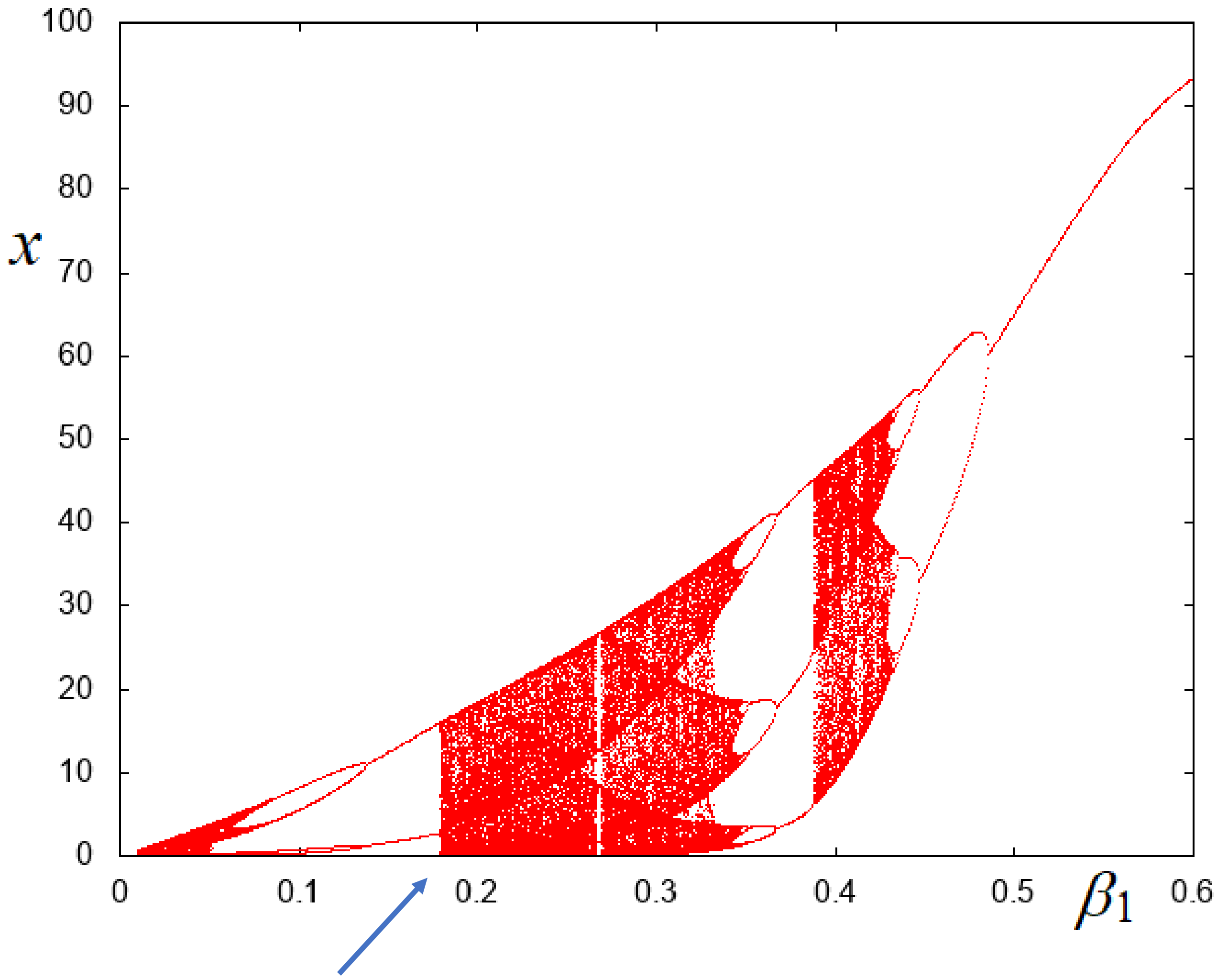
Recall that: $0 \leq \beta_2 \leq \beta_1 < 1$, $0 < \alpha < 1$, $A > 0, \sigma > 0$ $(\neq 1)$, $n > -1$, $0 \leq \delta \leq 1$, $\bar{y} \geq 0$

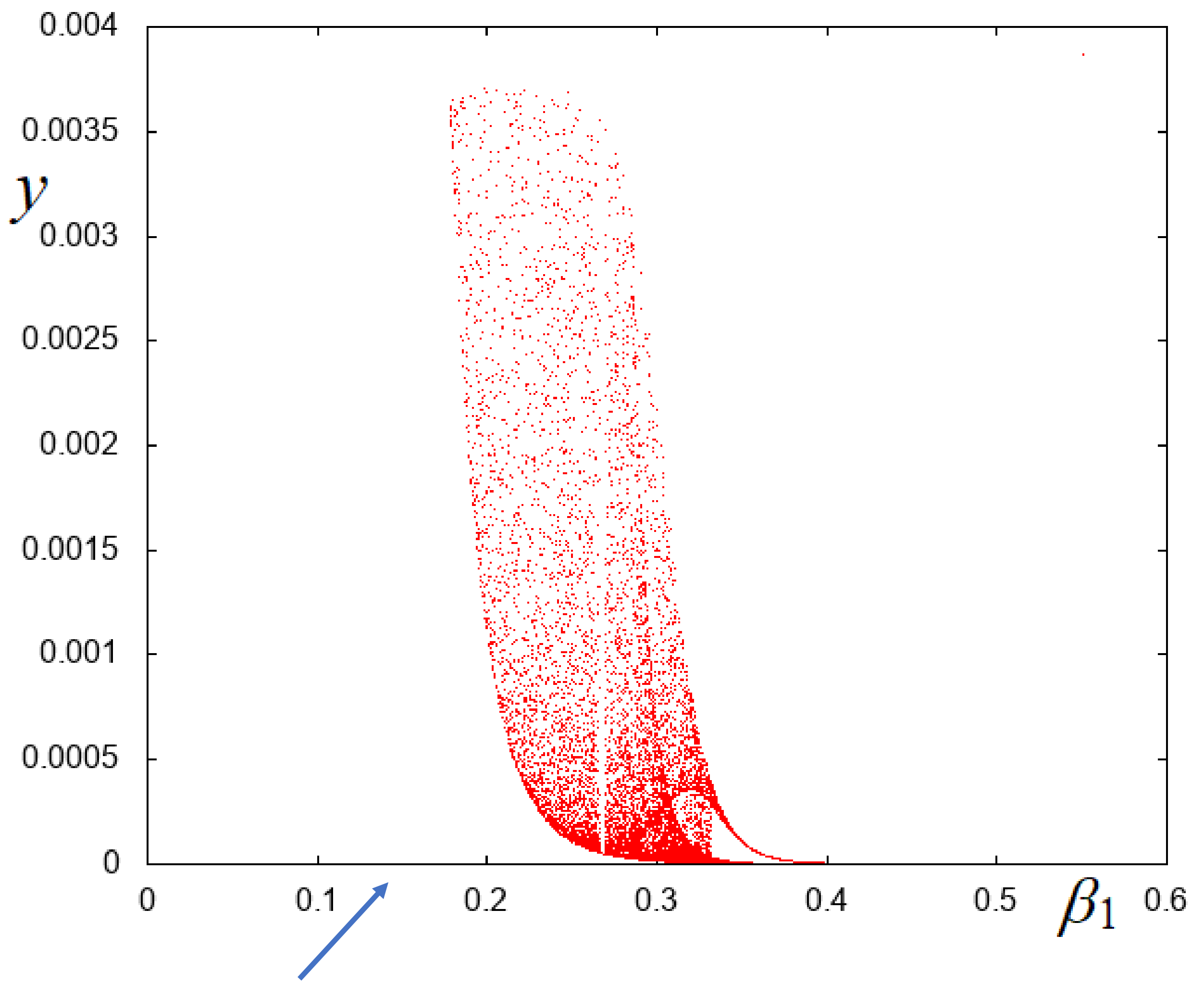
We fix $n = 0.01$, $\delta = 0.2$, $\alpha = 0.5$

and consider two parameter sets:

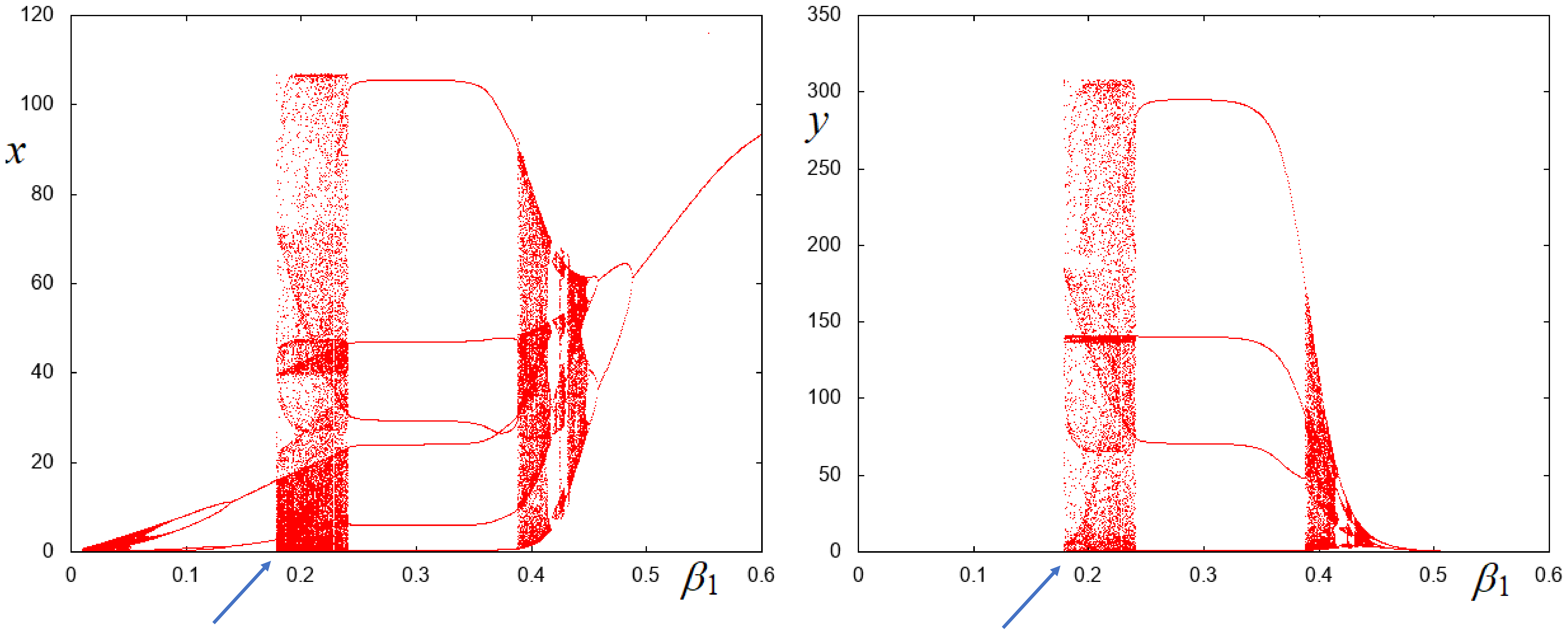
Set 1: $A = 20, \sigma = 25, \bar{y} = 40$

Set 2: $A = 10, \sigma = 12, \bar{y} = 10$



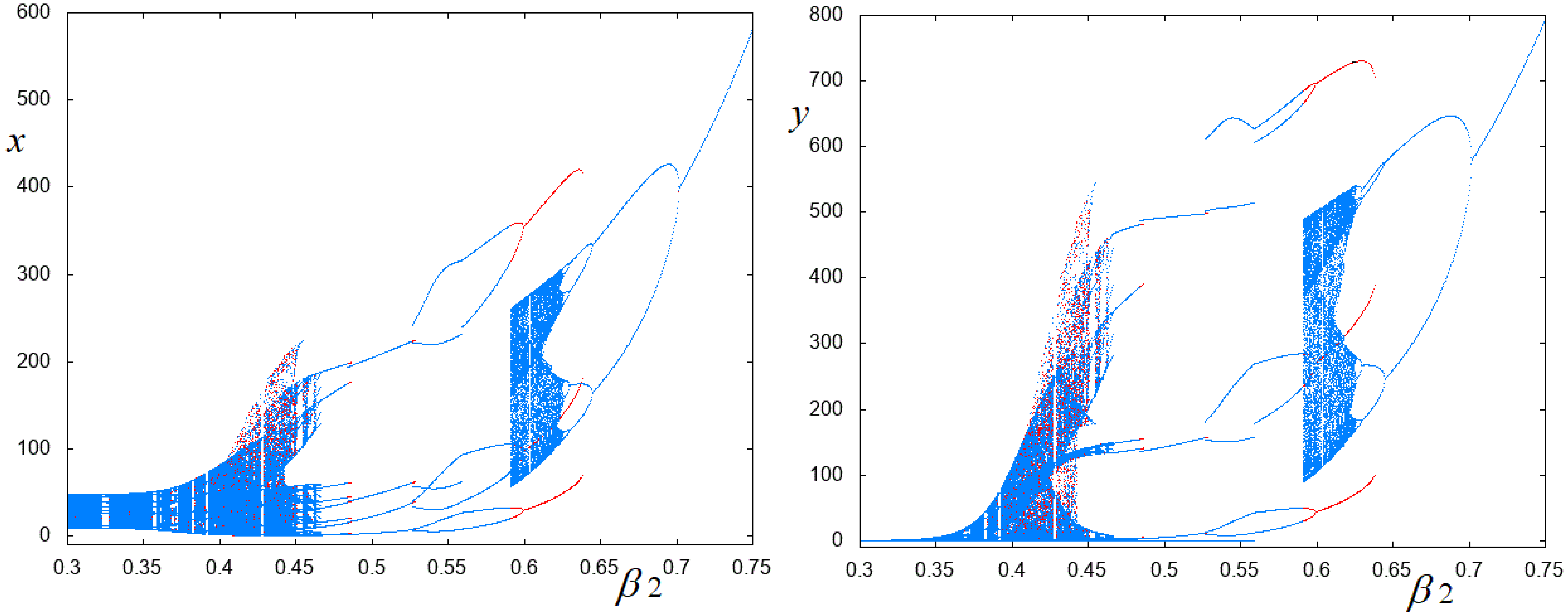


1D bifurcation diagrams w.r.t. β_1, for $\beta_2 = 0.2$, set 1.



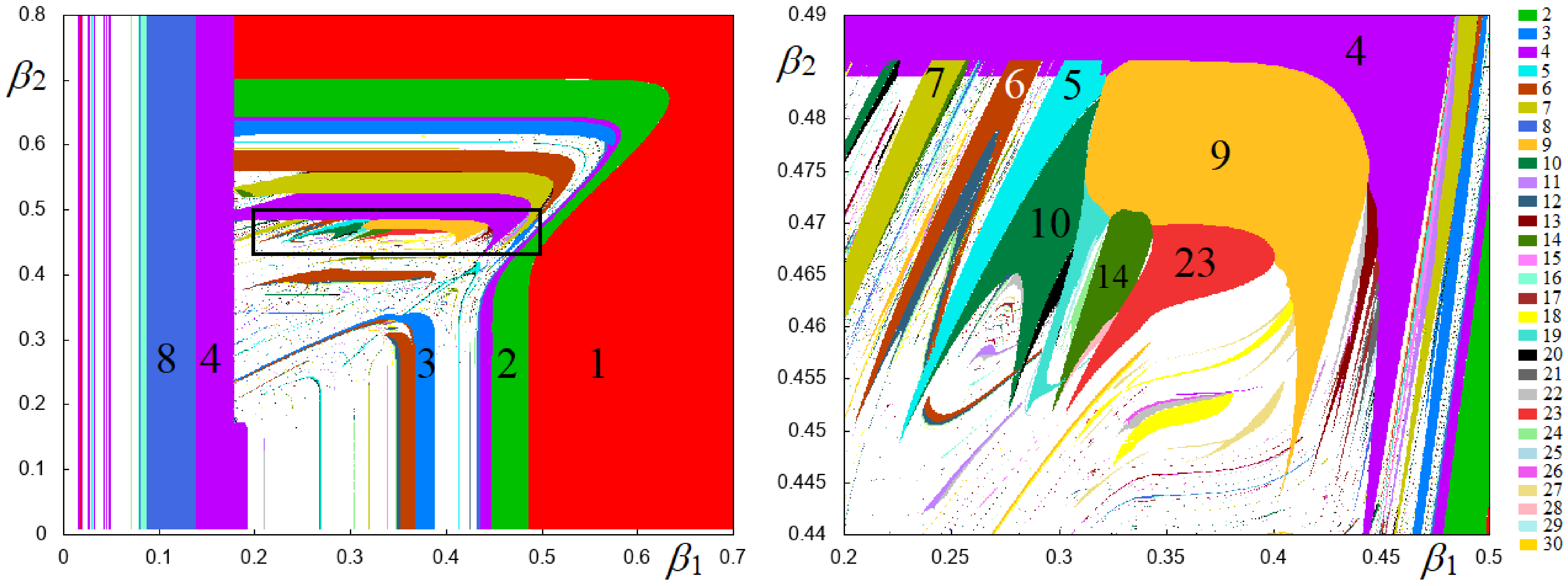
1D bifurcation diagrams w.r.t. β_1, for $\beta_2 = 0.4$, set 1.

Set 1, β_1=0.4



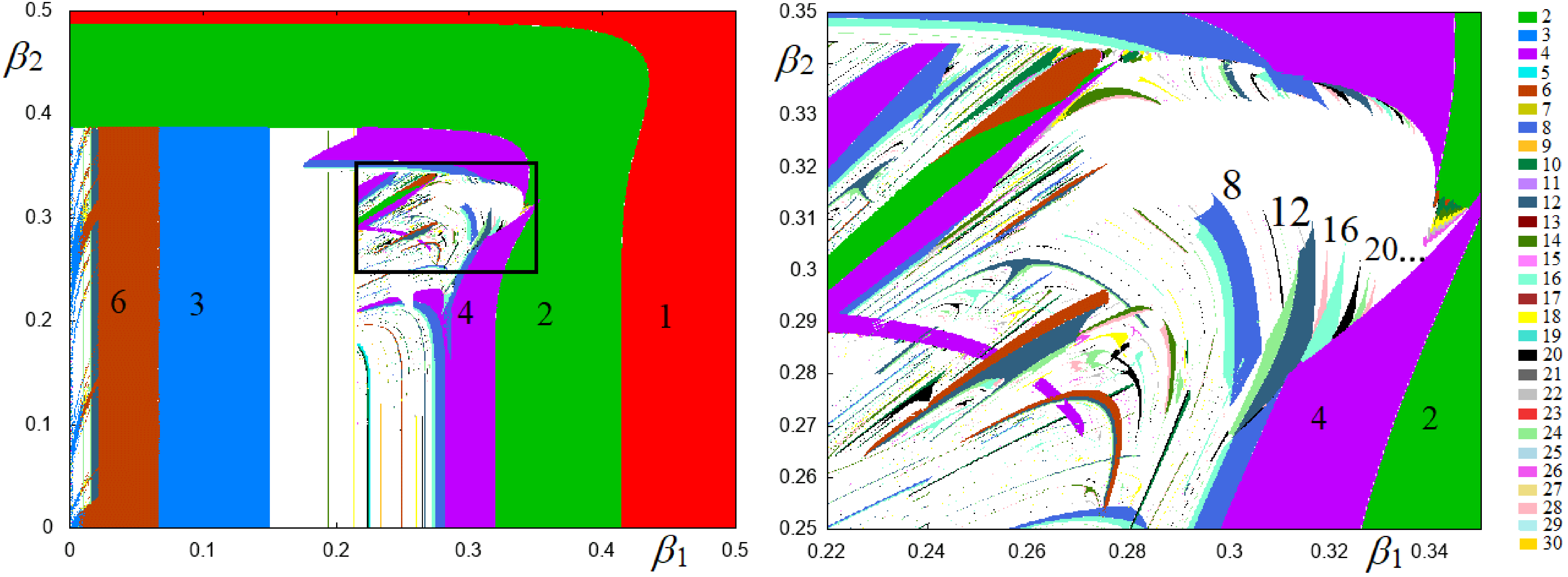
1D bifurcation diagrams w.r.t. β_2, with two different initial conditions

$n=0.01,\ \delta=0.05,\ \alpha=0.5,\ \sigma=25,\ A=20,\ \bar{y}=40$

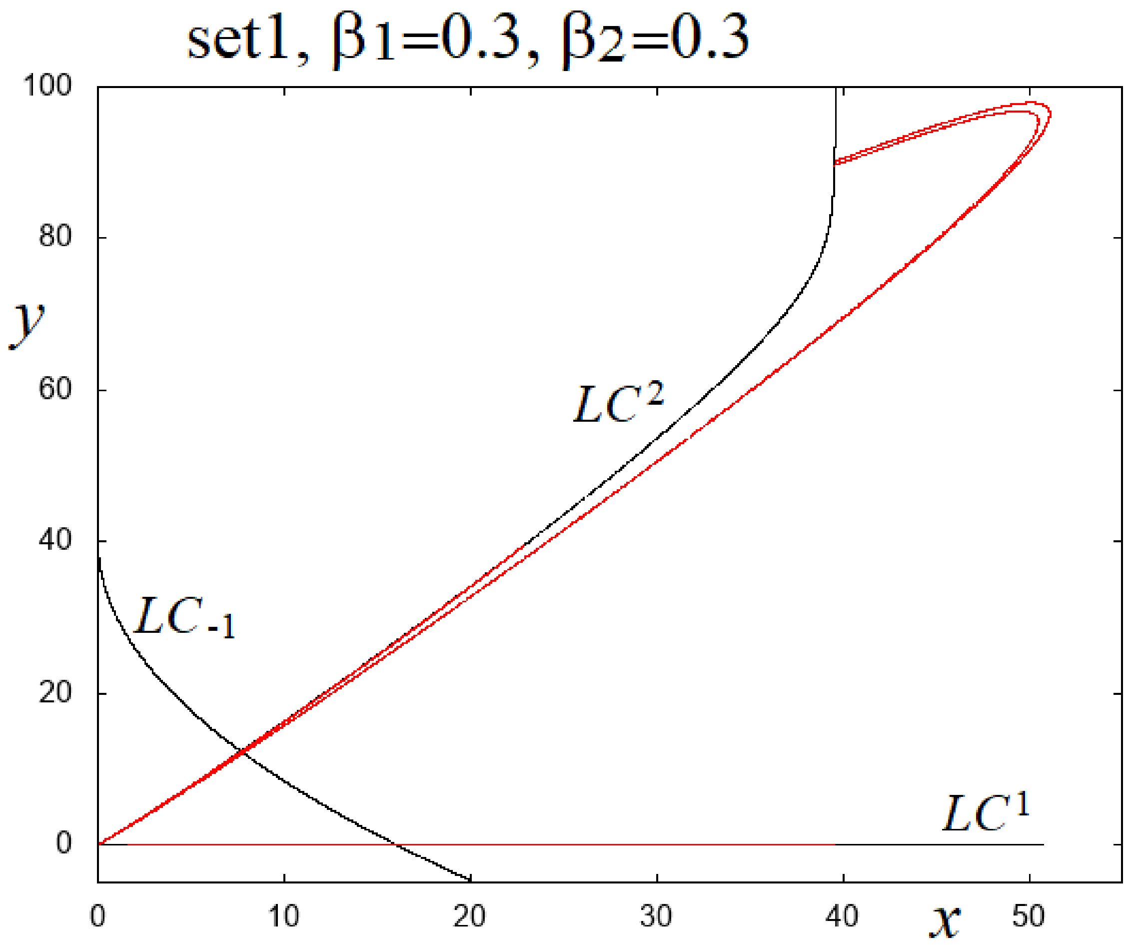


2D bifurcation diagram w.r.t β_1 *and* β_2, for parameter set 1; enlargement

$n=0.01,\ \delta=0.05,\ \alpha=0.5,\ \sigma=12,\ A=10,\ \overline{y}=10$

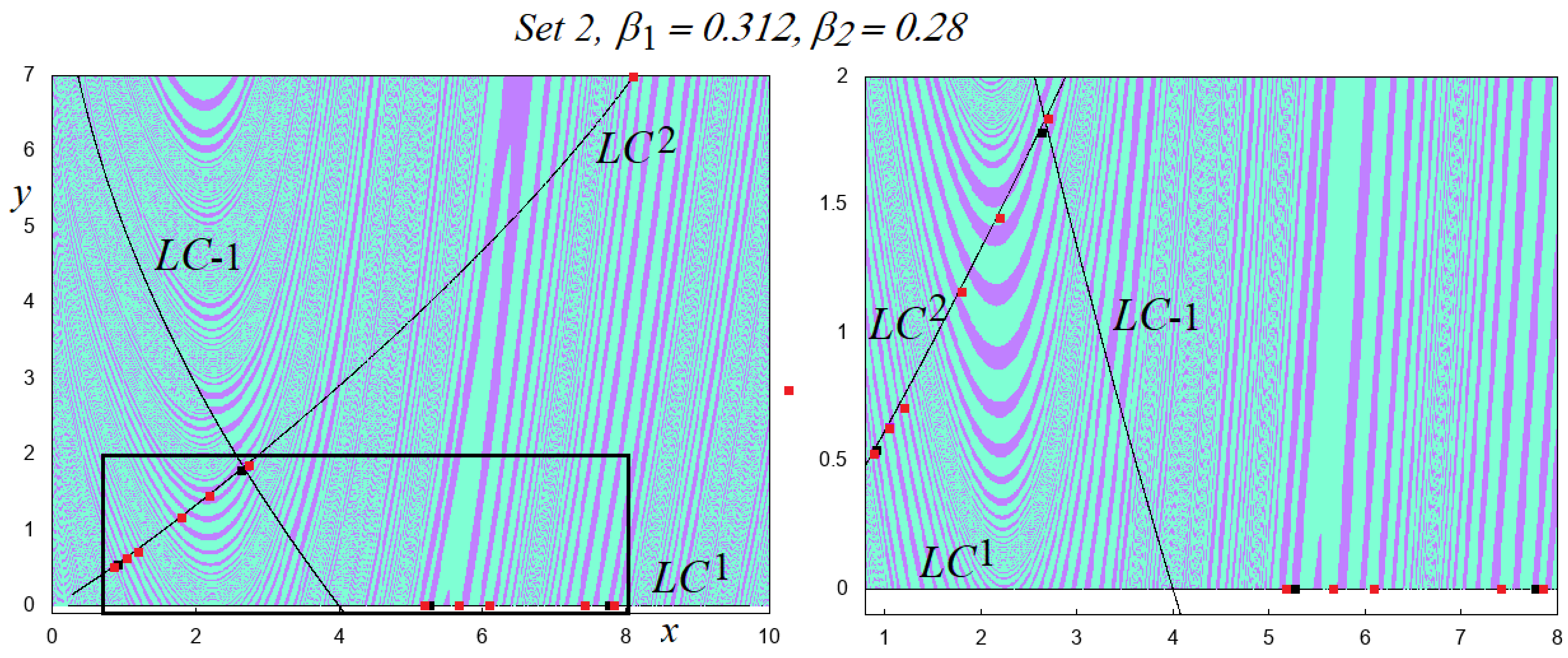


2D bifurcation diagram w.r.t β_1 *and* β_2, for parameter set 2; enlargement



$$LC_{-1}: y = \bar{y} - (1-\alpha)Ax^{\alpha}$$
$$LC_1 = F_1(LC_{-1}) = 0$$
$$LC_2 = F_2(LC_{-1})$$

Discontinuity line LC_{-1} and its images LC^1 and LC^2. In red: two-piece chaotic attractor (one piece belonging to y=0). Set 1

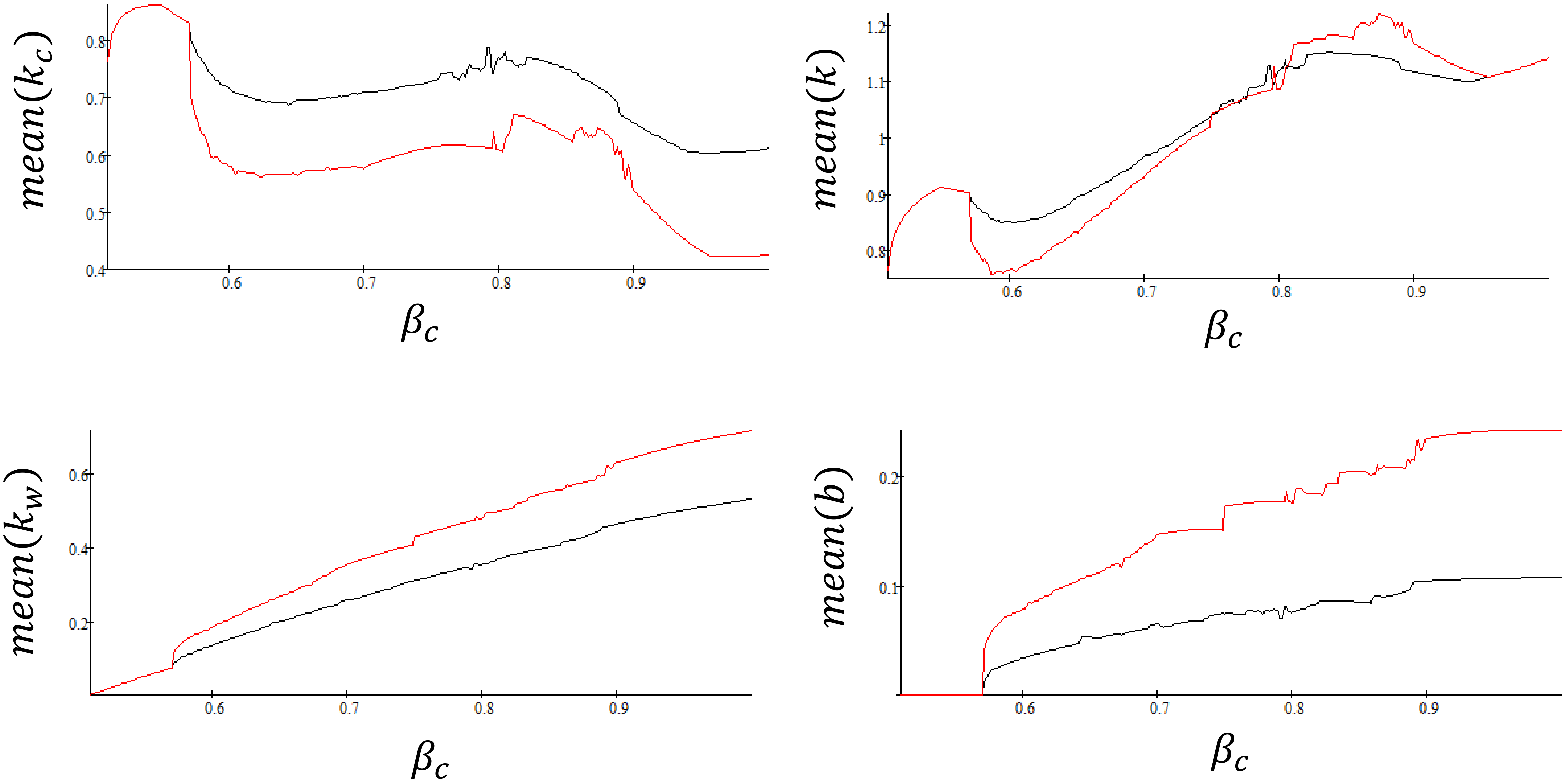


Coexisting 4-cycle (with 2 points on the y-axis) and 12-cycle (with 5 points on the y-axis). Parameter set 2.

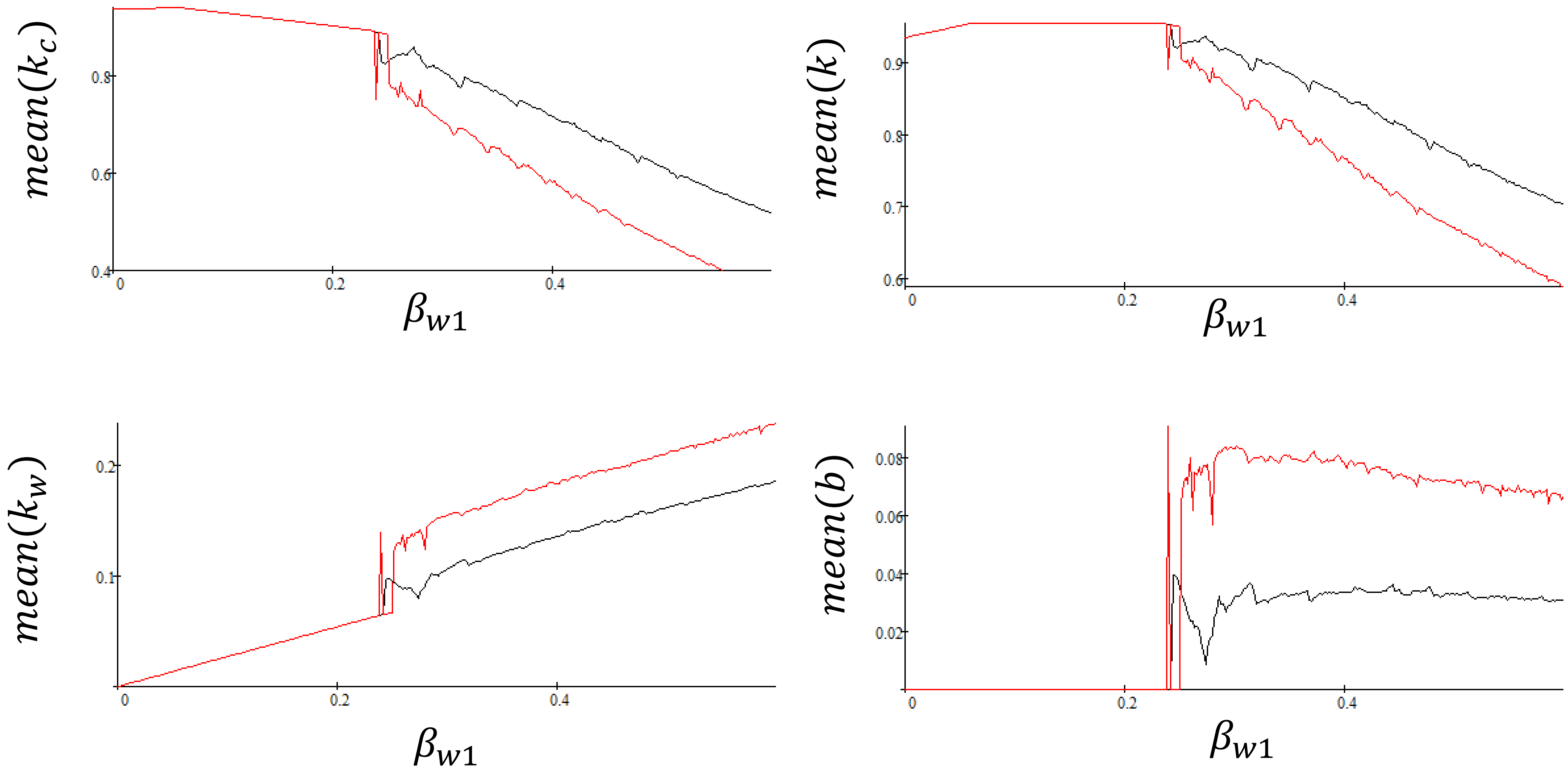
Full model
(preliminary results via simulations)

- Capitalists’ capital is positive
- Utility function is logarithmic
- Production function is CES

Black line $\beta_{w2} = 0.1$ Red line $\beta_{w2} = 0.2$



Black line $\beta_{w2} = 0.1$ Red line $\beta_{w2} = 0.2$



Final remarks

- We presented an overlapping generations model where two types of agents may co-exist – workers and capitalists – and where workers behaviour may have a discontinuity concerning their altruistic behaviour.
- We mostly focused on a simplified version of the model where capitalists are not present (their capital is zero).

- In summary, for the simplified 2-D model we found that:
- Increasing β_1 (or β_2) has a stabilizing effect when β_1 (or β_2) is high.
- The discontinuity in agents behaviour impinges on the dynamics and has an impact on the (periodic or chaotic) attractors.
- We found evidence of period adding structures.
- Moreover:
- β_1 has a positive impact on *k* and a negative impact on *b*.
- Instead β_2 has a positive impact on both *k* and *b*.

- For the full 3-D model we found the following (preliminary) results via simulations:
- β_{w2} has a negative impact on k_c and a positive impact on k_w with an ambiguous effect on k.
- Thus it seems that a shift of workers behaviour towards more altruism has a positive impact on their share of capital.
- β_c and β_{w1} have similar impact on distribution (negative on k_c and positive on k_w) opposite on capital accumulation (positive and negative) and mostly dissimilar on bequests (positive and slightly negative when altruism is active). The last two results are very sensitive to parameter choices.

References

References

- Andreoni, J. (1989). Giving with impure altruism: Applications to charity and Ricardian equivalence. Journal of political Economy, 97(6), 1447-1458.
- Andreoni, J. (1990). Impure altruism and donations to public goods: A theory of warm-glow giving. The economic journal, 100(401), 464-477.
- Barro, R. J. (1974). Are government bonds net wealth?. Journal of political economy, 82(6), 1095-1117.
- Chen, H. J., Li, M. C., & Lin, Y. J. (2008). Chaotic dynamics in an overlapping generations model with myopic and adaptive expectations. Journal of Economic Behavior & Organization, 67(1), 48-56.
- Commendatore, P., & Palmisani, C. (2009). The Pasinetti-Solow Growth Model with Optimal Saving Behaviour: A Local Bifurcation Analysis. In Topics On Chaotic Systems: Selected Papers from CHAOS 2008 International Conference (pp. 87-95).
- De Nardi, M. (2004). Wealth inequality and intergenerational links. The Review of Economic Studies, 71(3), 743-768.
- Diamond, P. A. (1965). National debt in a neoclassical growth model. The American Economic Review, 55(5), 1126-1150.
- Foley, D. K. & Michl, T. R. (1999). Growth and distribution. Harvard University Press.
- Michl, T. R. (2009). Capitalists, workers, and fiscal policy: a classical model of growth and distribution. Harvard University Press.
- Michel, P., & de La Croix, D. (2000). Myopic and perfect foresight in the OLG model. Economics Letters, 67(1), 53-60.
- Pasinetti, L. L. (1962). Rate of profit and income distribution in relation to the rate of economic growth. The Review of Economic Studies, 29(4), 267-279.