Competition and cooperation between public and private sectors in environmental maintenance

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Outline

1. Brief Literature review;
2. The John and Pecchenino's model with environmental taxation;
3. The zero-maintenance case and the 1-D dynamic system;
4. The model with endogenous environmental impact;
5. Possible extensions and remarks.

Literature review

- An overlapping generations model of growth and the environment (John and Pecchenino, 1994);
- Short-lived agents and the long-lived environment (John et al., 1995);
- Optimal tax schemes and the environmental externality (Ono, 1996);
- Environmental sustainability, nonlinear dynamics and chaos (Zhang, 1999);
- Multiple attractors and nonlinear dynamics in an overlapping generations model with environment (Naimzada and Sodini, 2010);
- Maladaptation and global indeterminacy (Antoci et al., 2019).

The model (1)

We consider an OLG framework (Diamond, 1965) in which:

- The utility function of the representative agent is given by:

$$U(c_{t+1}, E_{t+1}) = \ln(c_{t+1}) + \eta \ln(E_{t+1});$$

  where $\eta > 0$ is the given discount factor;
- The individual works only in young age, supplying one unit of labour and earning a real wage w_t. She invests m_t^A for improvement in environmental quality and the government levies a tax on wage at the rate $0 < \tau < 1$. Then, the life-cycle budget constraints are:

$$C_{t+1} = (1 + R_{t+1} - \delta)s_t$$

$$(1 - \tau)w_t = s_t + m_t^A$$

$$C_{t+1}, m_t^A, s_t \geq 0.$$

The model (2)

- The environmental quality evolves according to John and Pecchenino (1994),

$$E_{t+1} = (1-b)E_t - \beta\, c_t + \gamma\,(m_t^A + \tau\, w_t);$$

  where $b \in (0,1)$ measures the autonomous decay of the environment while $\beta, \gamma > 0$ measure the negative impact of the consumption and the positive impact of the environmental improvement, respectively.
- A unique material good is produced by a representative firm using a Cobb-Douglas technology:

$$Y_t = F(K_t) = Ak_t^{\alpha}$$

  and the profit maximisation implies that

$$w_t = A(1-\alpha)K_t^{\alpha-1}$$

$$r_t = A\alpha\, K_t^{\alpha-1}$$

Existence of a Zero-maintenance manifold

- The maximisation of the utility function under the intertemporal budget constraint allows to obtain the optimal mix of saving and maintenance private expenditure. In particular,

$$m_A^* = \frac{((\alpha-1)((\tau-1)\eta+\tau)\gamma+\beta\alpha)Ak_t^{\alpha}-k(\delta-1)\beta+(b-1)E_t}{\gamma(\eta+1)}$$

where $\frac{\partial m_A^*}{\partial \tau} < 0$, from which we obtain that

$$\begin{cases} m_A^* > 0 & \textit{for} \quad E < \widetilde{E} \\ m_A^* = 0 & \textit{otherwise} \end{cases}$$

where

$$\widetilde{E} = \frac{((\alpha-1)((\tau-1)\eta+\tau)\gamma+\beta\,\alpha)A{K_t}^{\alpha}-K_t(\delta-1)\beta}{1-b}$$

The 1-D dynamical system in E_t

- In the case of positive priva expenditure ($m_A^* > 0$), the relationship

$$K_{t+1} = \frac{E_{t+1}}{\eta\gamma}$$

follows by the market clearing condition $K_{t+1} = s_t$. Then, similarly to Zhang (1999) we have to analyse the 1-D dynamic system

$$E_{t+1} = \frac{\eta}{\eta+1}\left[((1-\alpha)\gamma-\beta)\gamma\, A\left(\frac{E_t}{\delta\,\gamma}\right)^{\alpha} + ((1-b)\eta\gamma + (1-\delta)\beta)E_t\right]$$

Stationary Points: Existence and stability

By calculating the steady states of the system, we have

$$E_1^* = \left(\frac{\beta(\delta - 1) - \gamma(1 + b\eta)}{A(\beta\alpha - \gamma(1 - \alpha))} \right)^{\frac{1}{1-\alpha}} \eta\gamma, \qquad E_2^* = 0$$

Proposition

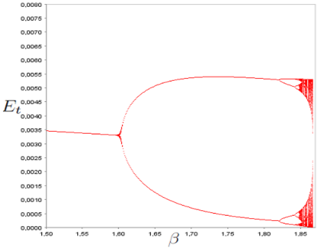
The map admits the fixed point E_1^* with $m_A^* > 0$ if and only if

$$\frac{\beta\alpha}{1 - \alpha} < \gamma < \frac{\beta}{(1 - \alpha)(b\delta(1 - \tau) - \tau)}$$

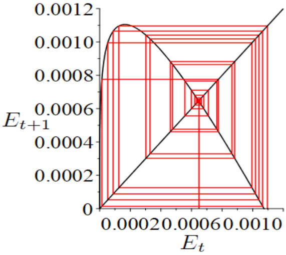
The fixed point E_1^* is not stable for every configuration of parameters and the map may undergo a Flip bifurcation.

Complex behaviour of the system for high levels of β

According to Zhang (1999), complex dynamics may emerge as the environmental impact of consumption (β) increases.



(a)



(b)

Figure: (a) $\alpha = 0.09, \tau = 0.4, \gamma = 1.33, A = 2, \eta = 0.03, b = 0.987, \delta = 0.03$; (b) the graph performed for $\beta = 1.86$.

The model with endogenous environmental impact of consumption

We consider now that (i) the environmental impact of consumption is not constant and (ii) the public expenditure for environment does not enter directly the environment dynamics but it is employed in reducing the impact. Then,

$$E_{t+1} = (1-b)E_t - \beta_t\, c_t + \gamma\, m_t^A$$

as in the previous specification, agents in economies with little capital or with high environmental quality may choose not to engage in environmental maintenance. Then,

$$m_A^* > 0 \qquad \textit{for} \quad E < \tilde{\tilde{E}}$$

where $\tilde{\tilde{E}} = \frac{((\alpha-1)((\tau-1)\eta+\tau)\gamma+\beta_t\,\alpha)AK_t{}^{\alpha} - K_t(\delta-1)\beta_t}{1-b}$.

Two different specifications for β_{t+1}

We introduce the following specification for the dynamics of the environmental impact of consumption

$$\beta_{t+1} = \beta_t + \left(\frac{\beta_0}{1+\phi_i} - \beta_t \right) \phi \qquad \text{with} \qquad i = 1, 2$$

where $\phi_1 = \tau\, w_t$ and $\phi_2 = \frac{\tau\, w_t}{Y_t}$ (alternative approaches) and $\beta_0 > 0$ is the "natural level" of environmental impact.

The dynamic system with ϕ_1

By assuming to have positive private contribution m_t^A, the direct relation $K_{t+1} = \frac{E_{t+1}}{\eta\gamma}$ continues to hold and then we have a 2-D dynamic system in E and β. In particular, we have:

$$\begin{cases} E_{t+1} = \frac{\eta}{\eta+1}\left(A((\alpha-1)(\tau-1)\gamma - \beta_t\,\alpha)\left(\frac{E_t}{\eta\,\gamma}\right)^{\alpha} + \frac{E_t\,(\delta-1)\beta_t}{\eta\,\gamma} - (b-1)E_t\right) \\ \beta_{t+1} = \beta_t - \left(\frac{\beta_0}{(1-\tau\,A(\frac{Et}{\eta\,\gamma})^{\alpha}(\alpha-1))} - \beta_t\right)\tau\,A(\frac{E_t}{\eta\,\gamma})^{\alpha}(\alpha-1) \end{cases} \quad (1)$$

The dynamic system with ϕ_1 (2)

By analysing the system (1), we are not able to find fixed points in a closed form and (at this stage) we have proved the existence of couples (E^*, β^*) solving the system only numerically.
We report the general form of the equilibrium:

$$E^* = \left(\frac{\beta_t(\delta - 1) - \gamma(1 + b\eta)}{A(\beta_t\alpha - \gamma(1 - \alpha))} \right)^{\frac{1}{1-\alpha}} \eta\gamma \qquad \beta^* = \frac{\beta_0}{1 - \tau A(\frac{Et}{\eta\gamma})^{\alpha}(\alpha - 1)}$$

The dynamic system with ϕ_2

$$
\begin{cases}
E_{t+1} = \frac{\eta}{\eta+1}\left(A((\alpha-1)(\tau-1)\gamma - \beta_t\,\alpha)\left(\frac{E_t}{\eta\,\gamma}\right)^{\alpha} + \frac{E_t\,(\delta-1)\beta_t}{\eta\,\gamma} - (b-1)E_t\right) \\
\beta_{t+1} = \beta_t - \left(\frac{\beta_0}{(1-\tau\,(\alpha-1))} - \beta_t\right)\tau\,(\alpha-1)
\end{cases}
\tag{2}
$$

The dynamic system with ϕ_2 (2)

The dynamic system (2) is triangular and it is possible find fixed points in closed form

$$E^* = \left(\frac{(-1+\tau\ (\alpha-1))\,(b\eta+1)\,\gamma 1+(\delta-1)\,B0}{((\alpha-1)\,(\tau-1)\,(\alpha\,\tau-\tau-1)\,\gamma 1+B0\,\alpha)\,A} \right)^{\frac{1}{1-\alpha}}, \quad 0$$

$$\beta^* = \frac{\beta_0}{1+\tau 1-\alpha)}$$

Complex behaviour as productivity A varies

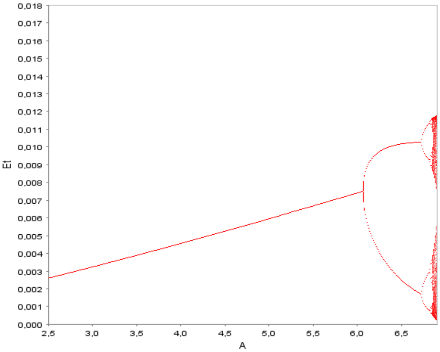


Figure: (a)
$\alpha = 0.16, \tau = 0.75, \gamma = 1.33, A = 2, \eta = 0.034, b = 0.987, \delta = 0.003, \beta_0 = 0.001.$

we have that $J_{11}(E^*, \beta_t) < -1$ for an appropriate parameters configuration, then Flip bifucations may occur.

Thank you