Business cycles and government bond purchase by central banks in monetary union

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Significance of Research

Purchasing government bond of a certain country can stabilize unsynchronized business cycles of two countries in a monetary union.

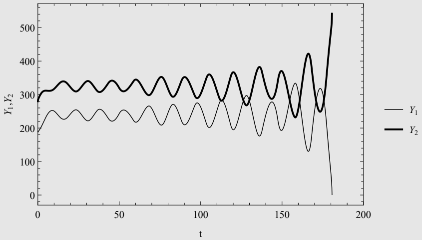


Figure 1: Before purchase

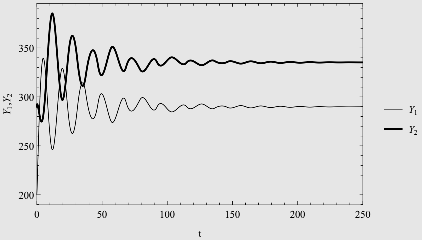


Figure 2: After purchase in country 1

Outline

Introduction

Background

The Euro area brought the euro crisis to an end by declaring an introduction of **Outright Monetary Transactions** (OMT).

While the Euro area is not an **optimum currency area** (OCA), the Euro area exists today.

- Basis: Mundell (1961), McKinnon (1963), and Kenen (1969)
- Survey: Mongelli (2002), Baldwin and Wyplosz (2015), and De Grauwe (2018)

The theory of OCA focus on the **synchronization of business cycles** in a monetary union.

- Frankel and Rose (1996), Gächter et al. (2012), and De Grauwe and Ji (2016, 2017)

Problems

Several studies have proved that **the synchronization of business cycles** is increased before introduction of the euro, while it is decreased after the introduction.

- Before: Altavilla (2004), Camacho et al. (2006), and Darvas and Szapáry (2008),
- After: Weyerstraß et al. (2011) and Gächter et al. (2012)

However, what seems to be lacking is an analysis on how **the purcahase of government bonds by central banks in a monetary uninon** affects **the stability and the synchronization** of business cycles.

It is important to analyze the combination of the theory of OCA and the stability and the synchronization of business cycles.

Purpose and method

Purpose

- To discuss whether buying a government bond of a country in a monetary union stabilize the business cycles of several countries in a monetary union.
- To analyze whether purchasing a government bond of a coutnry stabilize the business cycles even if the business cycles are not synchronized.

Method

- Kaldorian two-country model with a monetary union and imperfect capital mobility
- Relevant research: Asada, Inaba and Misawa (2001), Asada, Chiarella, Flaschel and Franke (2003), and Asada (2004)

Model

Assumption

Assumption 1

$$E = E^e = \bar{E} = 1 \tag{1}$$

Assumption 2

$$p_i = 1 \tag{2}$$

The subscript i ($i = 1, 2$): the index number of a country

E: exchange rate

E^e: expected exchange rate of the near future

p_i: price level of country i

Government bond

$$B_i = B_i^i + B_i^j + \boldsymbol{\theta_i}, \tag{3}$$

$$\dot{B}_i = \dot{B}_i^i + \dot{B}_i^j + \dot{\boldsymbol{\theta}}_{\boldsymbol{i}} \tag{4}$$

B_i: outstanding nominal government bonds of country i

B_i^i: outstanding nominal government bonds of country i held by a private sector in country i

B_i^j: outstanding nominal government bonds of country i held by a private sector in country j

θ_i: **outstanding government bonds of coutry i held by the supranational central bank system that is included central banks of each country**

Budget constraint

$$\dot{B}_i = G_i + B_i r_i - T_i - \dot{\boldsymbol{\theta}}_{\boldsymbol{i}} \tag{5}$$

G_i: real government expenditure

r_i: nominal rate of interest

T_i: real income tax

Net exports, capital account, and total balance

$$J_1 + J_2 = 0, \tag{6}$$

$$Q_1 + Q_2 = 0, \tag{7}$$

$$A_1 + A_2 = 0, \tag{8}$$

$$A_i = J_i + Q_i \tag{9}$$

$$J_i = J_i(Y_i, Y_j)\ ;\ J^i_{Y_i} = \frac{\partial J_i}{\partial Y_i} < 0, J^i_{Y_j} = \frac{\partial J_i}{\partial Y_j} > 0, \tag{10}$$

J_i: real net exports

Q_i: real capital account balance

A_i: real total balance

Y_i: real net national income

Money supply

$$M = M_1 + M_2, \tag{11}$$

$$\dot{M} = \dot{\boldsymbol{\theta}}_{\mathbf{1}} + \dot{\boldsymbol{\theta}}_{\mathbf{2}}, \tag{12}$$

$$\dot{M}_i = A_i + \dot{\boldsymbol{\theta}}_{\boldsymbol{i}}, \tag{13}$$

$$M_i = L_i(Y_i, r_i)\ ;\ \frac{\partial L_i}{\partial Y_i} > 0, L^i_{r_i} = \frac{\partial L_i}{\partial r_i} < 0, \tag{14}$$

M: nominal money supply in the whole of monetary union

M_i: nominal money supply

L_i: demand for money

Capital account balance function

$$\dot{B}_1^2 = \boldsymbol{\beta}(r_1 - r_2)\ ;\ \beta > 0, \tag{15}$$

$$\dot{B}_2^1 = \boldsymbol{\beta}(r_2 - r_1), \tag{16}$$

$$\dot{B}_1^2 = -\dot{B}_2^1 = \boldsymbol{\beta}(r_1 - r_2), \tag{17}$$

$$B_1^2 + B_2^1 = \bar{D}, \tag{18}$$

$$\begin{aligned} Q_1 &= \dot{B}_1^2 - \dot{B}_2^1 + r_2 B_2^1 - r_1 B_1^2 \\ &= \beta(r_1 - r_2) - \beta(r_2 - r_1) + r_2 B_2^1 - r_1 B_1^2 \\ &= 2\boldsymbol{\beta}(r_1 - r_2) + r_2 B_2^1 - r_1 B_1^2 \end{aligned} \tag{19}$$

β: **degree of mobility of international capital flows**

Disequilibrium quantity adujustment process, consumption, and tax

$$\dot{Y}_i = \alpha_i[C_i + I_i + G_i + J_i - Y_i] \ ; \ \alpha_i > 0, \tag{20}$$

$$C_i = c_i(Y_i + r_i B_i^i + r_j B_j^i - T_i) + C_{0i} \ ; \ 0 < c_i < 1, C_{0i} \geq 0, \tag{21}$$

$$T_i = \tau_i(Y_i + r_i B_i^i + r_j B_j^i) - T_{0i} \ ; \ 0 < \tau_i < 1, T_{0i} \geq 0, \tag{22}$$

Y_i: real net national income

C_i: real private consumption expenditure

α: adjustment speed of the goods market

c_i: marginal propensity to consume

C_{0i}: basic consumption

I_i: real net private investment expenditure

G_i: real government expenditure

τ_i: marginal tax rate

T_{0i}: negative income tax (or basic income)

Investment, capital stock, and government expenditure

$$I_i = I_i(Y_i, K_i, r_i) \ ; \ I^i_{Yi} = \frac{\partial I_i}{\partial Y_i} > 0, I^i_{K_i} = \frac{\partial I_i}{\partial K_i} < 0, I^i_{r_i} = \frac{\partial I_i}{\partial r_i} < 0, \tag{23}$$

$$\dot{K}_i = I_i, \tag{24}$$

$$G_i = G_{0i} + \boldsymbol{\gamma_i}(\bar{Y}_i - Y_i) \ ; \ \gamma_i > 0, \tag{25}$$

K_i: real capital stock

G_{0i}: basic government expenditure

γ_i: **degree of counter-cyclical fiscal policy**

$\bar{Y}_i$: the level of real national income that a government determine the counter-cyclical government expenditure (this is not necessarily natural output)

Assumption

Assumption 3

$$\dot{\theta}_1 = \dot{\theta}_2 = 0 \tag{26}$$

$$\bar{M} = M_1 + M_2 \tag{27}$$

Therefore, we transform Eqs. (5), (12) and (13) into the following equations.

$$\dot{B}_i = G_i + B_i r_i - T_i, \tag{28}$$

$$\dot{M} = 0 \tag{29}$$

$$\dot{M}_i = A_i \tag{30}$$

Eight-dimensional dynamical system (i)

$$\begin{aligned}
\dot{Y}_1 =& \alpha_1[\{c_1(1-\tau_1)-1\}Y_1 \\
&+ c_1(1-\tau_1)\{(B_1-\boldsymbol{\theta}_1-B_1^2)r_1(Y_1,M_1)+(\bar{D}-B_1^2)r_2(Y_2,\bar{M}-M_1)\} \\
&+ c_1T_{01}+C_{01}+G_{01}+\boldsymbol{\gamma_1}(\bar{Y_1}-Y_1)+I_1(Y_1,K_1,r_1(Y_1,M_1))+J_1(Y_1,Y_2)] \\
=& F_1(Y_1,K_1,B_1,B_1^2,Y_2,M_1;\alpha_1,\gamma_1,\theta_1), \qquad (31) \\
\dot{K}_1 =& I_1(Y_1,K_1,r_1(Y_1,M_1)) = F_2(Y_1,K_1,M_1), \qquad (32) \\
\dot{B}_1 =& G_{01}+\gamma_1(\bar{Y_1}-Y_1)+B_1r_1(Y_1,M_1) \\
&- \tau_1\{Y_1+(B_1-\boldsymbol{\theta_1}-B_1^2)r_1(Y_1,M_1)+(\bar{D}-B_1^2)r_2(Y_2,\bar{M}-M_1)\}+T_{01} \\
=& F_3(Y_1,B_1,B_1^2,Y_2,M_1;\gamma_1,\theta_1), \qquad (33) \\
\dot{B}_1^2 =& \boldsymbol{\beta}\{r_1(Y_1,M_1)-r_2(Y_2,\bar{M}-M_1)\} \\
=& F_4(Y_1,Y_2,M_1;\beta), \qquad (34)
\end{aligned}$$

Eight-dimensional dynamical system (ii)

$$
\begin{aligned}
\dot{Y_2} =& \alpha_2[\{c_2(1-\tau_2)-1\}Y_2 \\
&+ c_2(1-\tau_2)\{(B_2-\boldsymbol{\theta_2}-(\bar{D}-B_1^2))r_2(Y_2,\bar{M}-M_1)+B_1^2 r_1(Y_1,M_1)\} \\
&+ c_2T_{02}+C_{02}+G_{02}+\boldsymbol{\gamma_2}(\bar{Y_2}-Y_2)+I_2(Y_2,K_2,r_2(Y_2,\bar{M}-M_1)) \\
&- J_1(Y_1,Y_2)] \\
=& F_5(Y_1,B_1^2,Y_2,K_2,B_2,M_1;\alpha_2,\gamma_2,\theta_2), && (35) \\
\dot{K_2} =& I_2(Y_2,K_2,r_2(Y_2,M_1)) = F_6(Y_2,K_2,M_1), && (36) \\
\dot{B_2} =& G_{02}+\boldsymbol{\gamma_2}(\bar{Y_2}-Y_2)+B_2r_2(Y_2,M_1) \\
&- \tau_2\{Y_2+(B_2-\boldsymbol{\theta_2}-(\bar{D}-B_1^2))r_2(Y_2,M_1)+B_1^2r_1(Y_1,M_1)\}+T_{02} \\
=& F_7(Y_1,B_1^2,Y_2,B_2,M_1;\gamma_2,\theta_2), && (37) \\
\dot{M_1} =& J_1(Y_1,Y_2)+2\boldsymbol{\beta}\{r_1(Y_1,M_1)-r_2(Y_2,M_1)\}+(\bar{D}-B_1^2)r_2(Y_2,M_1) \\
&- B_1^2r_1(Y_1,M_1) \\
=& F_8(Y_1,B_1^2,Y_2,M_1;\beta) && (38)
\end{aligned}
$$

Jacobian matrix of the system of Eqs. (31)–(38)

$$J = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} & F_{15} & 0 & 0 & F_{18} \\ F_{21} & F_{22} & 0 & 0 & 0 & 0 & 0 & F_{28} \\ F_{31} & 0 & F_{33} & F_{34} & F_{35} & 0 & 0 & F_{38} \\ F_{41} & 0 & 0 & 0 & F_{45} & 0 & 0 & F_{48} \\ F_{51} & 0 & 0 & F_{54} & F_{55} & F_{56} & F_{57} & F_{58} \\ 0 & 0 & 0 & 0 & F_{65} & F_{66} & 0 & F_{68} \\ F_{71} & 0 & 0 & F_{74} & F_{75} & 0 & F_{77} & F_{78} \\ F_{81} & 0 & 0 & F_{84} & F_{85} & 0 & 0 & F_{88} \end{bmatrix} \tag{39}$$

Proposition 1

(i) Suppose that the parameter β is fixed at any level. Then, the equilibrium point of the system (31)–(38) is **locally stable** if at least one of the parameters **θ_1, θ_2, γ_1 and γ_2 is sufficiently large**.

(ii) Suppose that the parameter θ_1, θ_2, γ_1 and γ_2 are relatively small and inequalities $F_{11} > 0$ and $F_{55} > 0$ hold. Then, the equilibrium point of the system (31)–(38) is **locally unstable** if the parameter **β is sufficiently large**.

Numerical simulations

Simulations: assumption

Based on Asada (2004), assume the following parameter values.

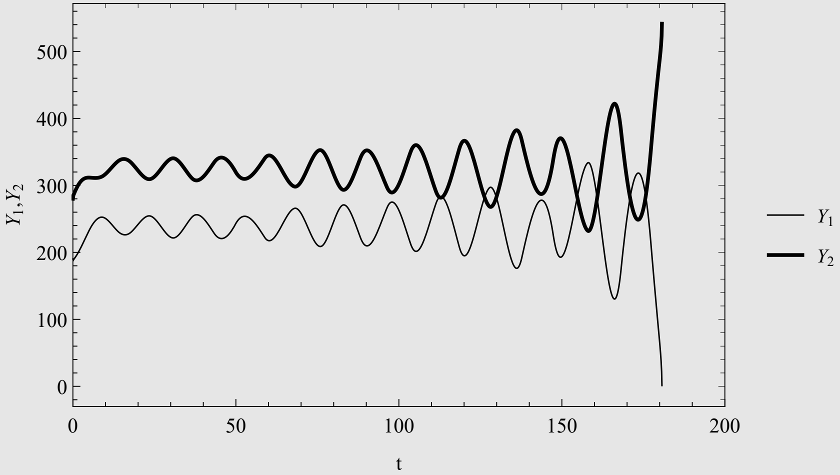
$$c_i = 0.8,\ \tau_i = 0.2,\ T_{0i} = 10,\ C_{01} = 20,\ C_{02} = 40,$$
$$G_{01} = 50,\ G_{02} = 60,\ \bar{M} = 600, \bar{Y}_i = 500,\ \bar{D} = 2,\ \alpha_i = 1,\ \beta = 5$$

$$r_i = 15\sqrt{Y_i} - M_i, \tag{40}$$
$$I_i = 20\sqrt{Y_i} - 0.3K_i - r_i, \tag{41}$$
$$J_1 = -0.35Y_1 + 0.25Y_2 \tag{42}$$

Compute the trajectories by selecting several values of θ_1, θ_2, γ_1 and γ_2 and the following initial conditions of the variables:

$$Y_1(0) = 190, K_1(0) = 1179, B_1(0) = 1.6, B_1^2(0) = 1,$$
$$Y_2(0) = 280, K_2(0) = 1351, B_2(0) = 0.9, M_1(0) = 280$$

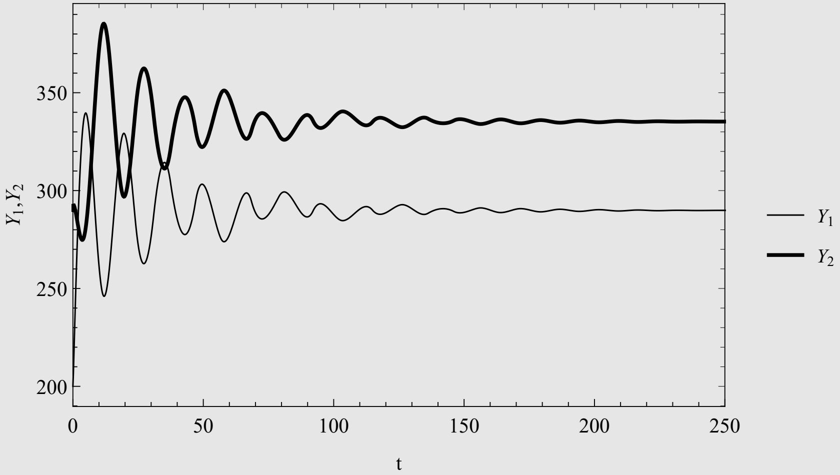
Business cycles under unstable economy



Note: $\beta = 5$, $\theta_i = 0$, $\gamma_i = 0.35$

Figure 3: High degree of capital movement and unsynchronized business cycles

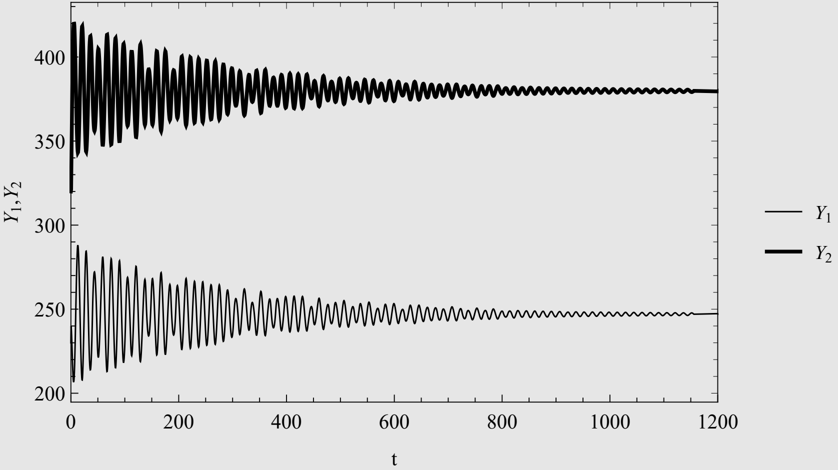
Purchasing government bond of country 1



Note: $\beta = 5$, $\theta_1 = 0.6$, $\theta_2 = 0$, $\gamma_i = 0.35$

Figure 4: Convergence by purchasing government bond of country 1

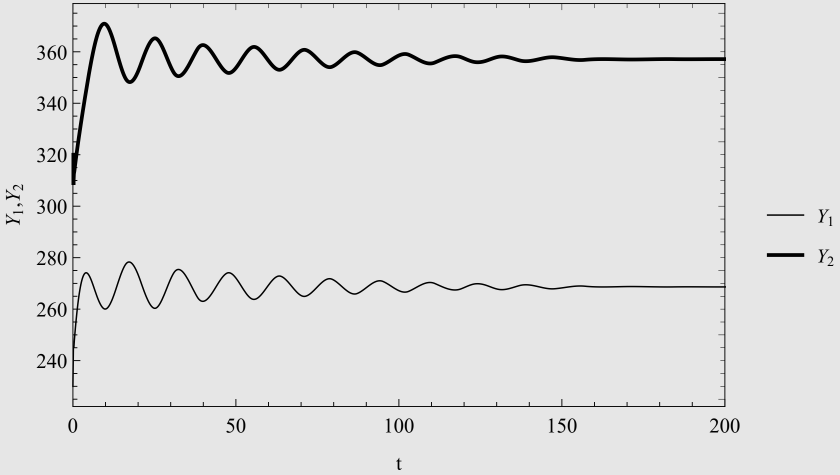
Purchasing government bond of country 2



Note: $\beta = 5$, $\theta_1 = 0$, $\theta_2 = 0.6$, $\gamma_i = 0.35$

Figure 5: Convergence by purchasing government bond of country 2

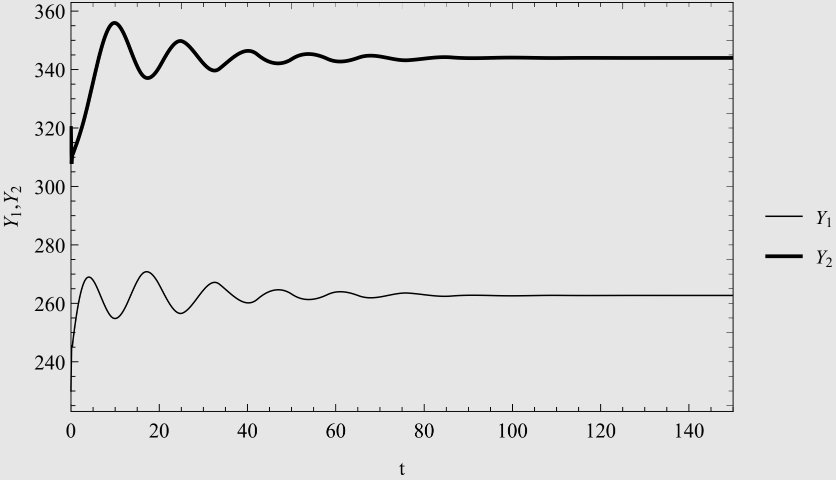
Purchasing government bond of both countries



Note: $\beta = 5$, $\theta_1 = 0.3$, $\theta_2 = 0.3$, $\gamma_i = 0.35$

Figure 6: Convergence by purchasing government bond of country 2

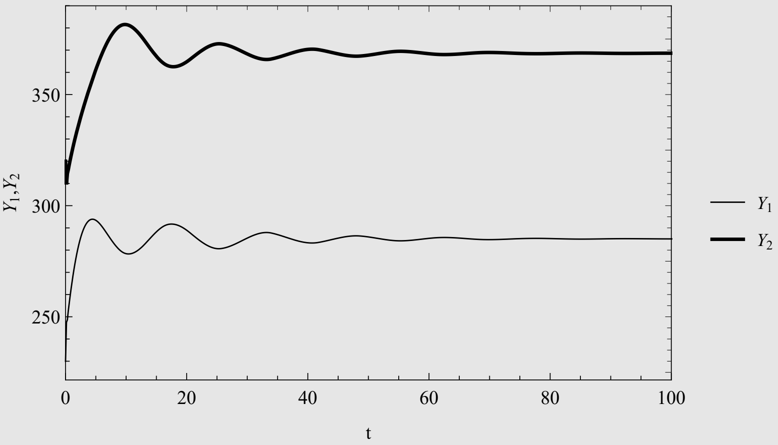
Counter-cyclical fiscal policy



Note: $\beta = 5$, $\theta_1 = 0$, $\theta_2 = 0$, $\gamma_i = 0.5$

Figure 7: Counter-cyclical fiscal policy can stabilize business cycles fluctuations

Purchasing government bond of both countries and counter-cyclical fiscal policy



Note: $\beta = 5$, $\theta_1 = 0.3$, $\theta_2 = 0.3$, $\gamma_i = 0.5$

Figure 8: Counter-cyclical fiscal policy can stabilize business cycles fluctuations

Results of simulations

Purchasing government bonds stabilize business cycles.

Purchasing a government bonds stabilize business cycles in a monetary union even if business cycles are not synchronized.

The combination of buying government bonds and a counter-cyclical fiscal policy stabilize business cycles more rapidly.

Conclusion

Conclusion

A policy such as OMT that buy a bond of a country in a crisis contributes to the stabilization of business cycles.

While the synchronization of business cycles is attracting attention in the theory of OCA, it is important that the purchase of government bonds by central banks can stabilize the cycle even if they are not synchronized.

One element for the euro area to become the OCA was acquired through measures against the euro crisis.

Future plan

Analyze the stability of the business cycle using a model with variable prices, in order to consider a policy aimed at influencing prices, such as quantitative easing.

Thank you very much for your kind attention.

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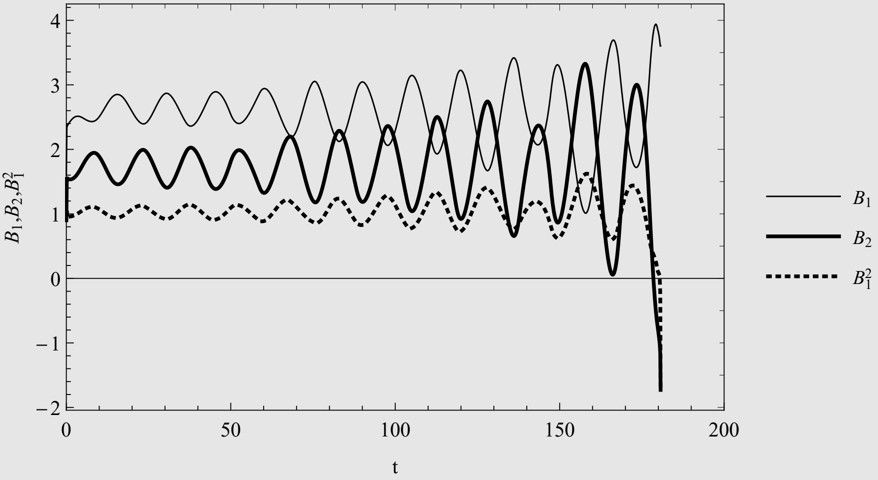
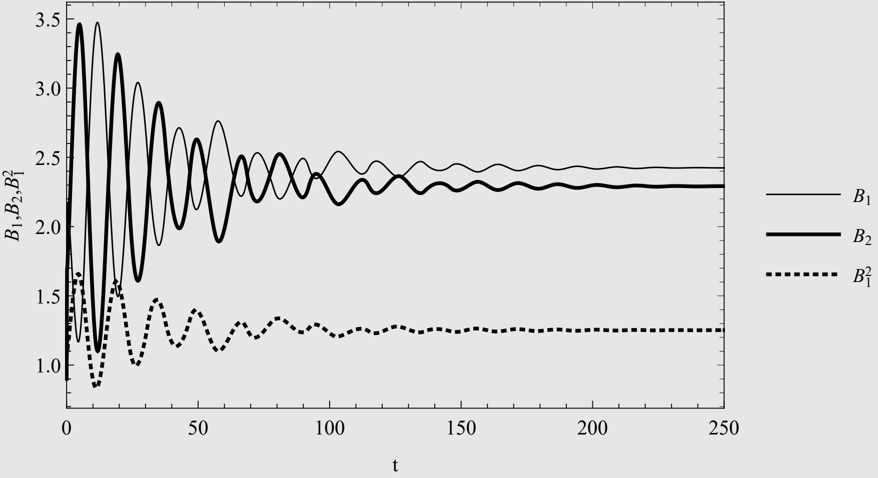
Business cycles of Unstable economy



Note: $\beta = 5$, $\theta_i = 0$, $\gamma_i = 0$

Figure 9: Not synchronized business cycles

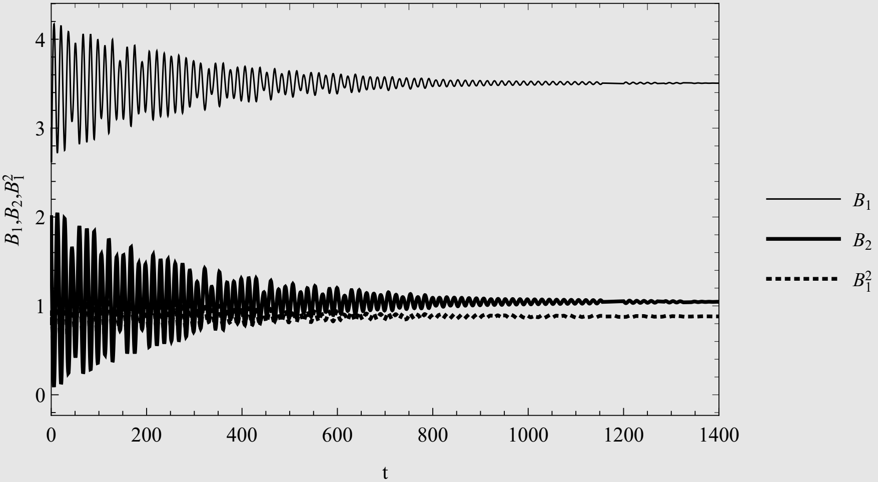
Purchasing government bond of country 1



Note: $\beta = 5$, $\theta_1 = 0.6$, $\theta_2 = 0$, $\gamma_i = 0.35$

Figure 10: Convergence by purchasing government bond of country 1

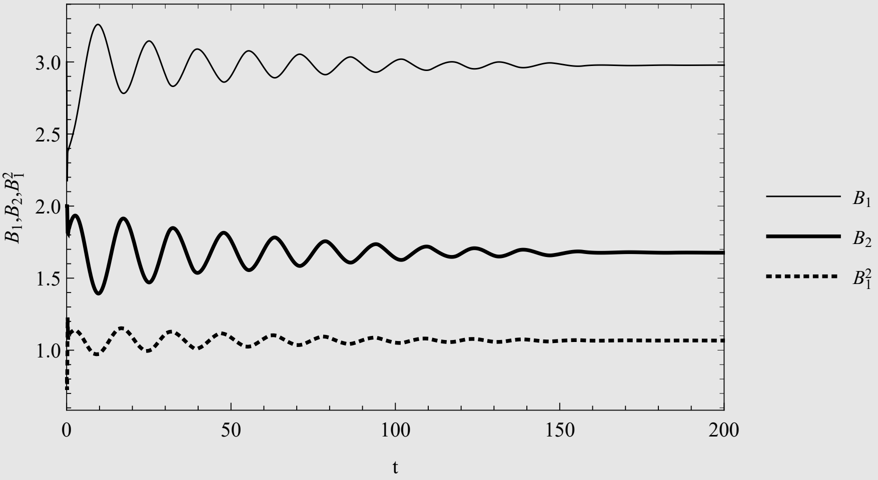
Purchasing government bond of country 2



Note: $\beta = 5$, $\theta_1 = 0$, $\theta_2 = 0.6$, $\gamma_i = 0.35$

Figure 11: Convergence by purchasing government bond of country 2

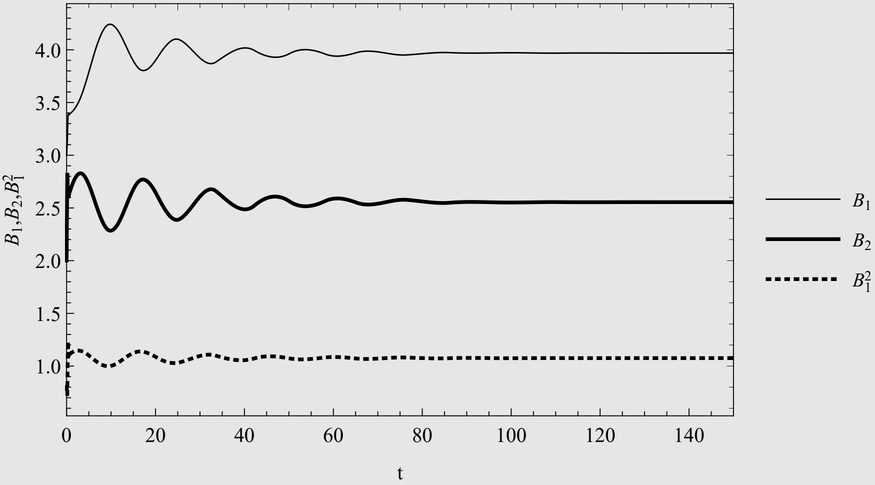
Purchasing government bond of both countries



Note: $\beta = 5$, $\theta_1 = 0.3$, $\theta_2 = 0.3$, $\gamma_i = 0.35$

Figure 12: Convergence by purchasing government bond of country 2

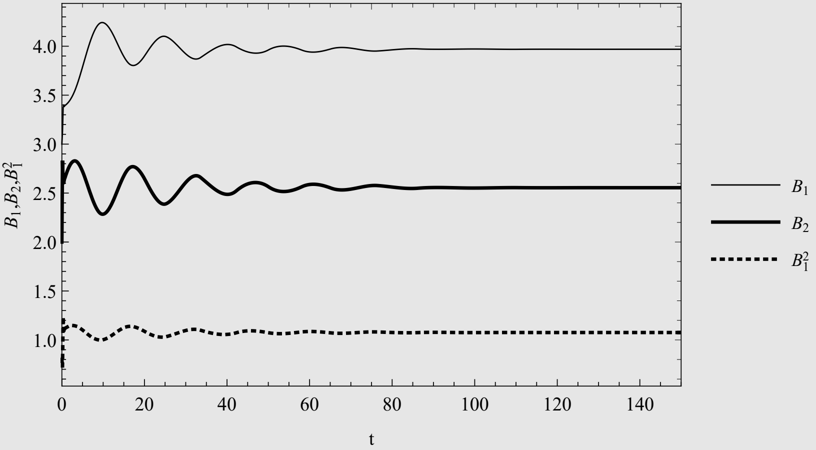
Counter-cyclical fiscal policy



Note: $\beta = 5$, $\theta_1 = 0$, $\theta_2 = 0$, $\gamma_i = 0.5$

Figure 13: Counter-cyclical fiscal policy can stabilize business cycles fluctuations

Purchasing government bond of both countries and counter-cyclical fiscal policy



Note: $\beta = 5$, $\theta_1 = 0.3$, $\theta_2 = 0.3$, $\gamma_i = 0.5$

Figure 14**:** Counter-cyclical fiscal policy can stabilize business cycles fluctuations